Blast from the Past - The Derivative of $\ln(x)$

Let $y = \ln(x)$.

1. (a) Solve this equation for $x$.

   (b) Use implicit differentiation to find $\frac{dy}{dx}$ in terms of $y$.

   (c) Use part (a) and (b) to show that $\frac{d}{dx} \ln(x) = \frac{1}{x}$.

Some Stuff from Last Time...

Last time, we looked at inverse trig functions. Today, we will differentiate them...

2. (a) What are the domain and range of $\arctan x$?

   (b) If $y = \arctan x$, then ________ = $x$.

Differentiating...

3. (a) Use implicit differentiation on the equation in question 2 to find $\frac{dy}{dx}$ in terms of $y$.

   (b) Starting from the identity $\sin^2 y + \cos^2 y = 1$, show that $\tan^2 y + 1 = \sec^2 y$.

   (c) Show that $\frac{d}{dx} \arctan x = \frac{1}{1+x^2}$.
4. Use a similar process to the above, this time using the identity \( \sin^2 y + \cos^2 y = 1 \) directly to find the derivatives of \( \sin^{-1} x \) and \( \cos^{-1} x \).

Record your results here:

\[
\frac{d}{dx} \sin^{-1} x = \phantom{0} \quad \frac{d}{dx} \cos^{-1} x = \phantom{0}
\]

5. Find the derivatives of the functions below:

(a) \( g(y) = \sin(\arcsin y) \)

(b) \( p(t) = t \sin^{-1} t \)
6. At what value(s) of \( x \) does the graph of \( \sin^{-1} x \) have vertical tangents?
   (Note: show this using the derivative of \( \sin^{-1} x \), but recall that we also stated the answer last time when we drew the graph of \( \sin^{-1} x \). Can you recall how we got there?)

7. Use the derivative of \( \tan^{-1} x \) to show that \( \tan^{-1} x \) has horizontal asymptotes as \( x \to \infty \) and as \( x \to -\infty \). (Hint: If a function has a horizontal asymptote, what must be true of its derivative as \( x \to \infty \) or \( -\infty \)?)

8. Use a linear approximation to estimate \( \arctan(1.1) \). (Hint: What is \( \arctan(1) \)?) Is your approximation an overestimate of the true value of \( \arctan(1.1) \) or an underestimate of it?