Make sure you are computing in radians. We will rarely use degrees from now on.

**Graphs of** $\sin(x)$ **and** $\cos(x)$

Since $\sin(x)$ and $\cos(x)$ are defined by using our unit circle, they repeat themselves every $\pi$. In other words:

- $\sin(x) = \sin(x + \pi) = \sin(x + \pi) = \ldots$ and
- $\cos(x) = \cos(x + \pi) = \cos(x + \pi) = \ldots$

**Question** Use Wolfram Alpha to draw the graphs of $\sin(x)$ and $\cos(x)$ below (on the same set of axes), with domain $[-2\pi, 2\pi]$ (You can literally type ‘graph of $\sin(x)$ from -2pi to 2pi’ into Wolfram Alpha):

The **Tangent Function**

We defined $\tan \theta = \frac{\text{OPP}}{\text{adj}} = \frac{\sin \theta}{\cos \theta}$. Using Wolfram Alpha, draw the graph of $\tan x$ with domain $[-2\pi, 2\pi]$ below:
Questions

1. What is the period of tan \( x \)?

2. For what values of \( x \) is tan \( x \) positive? Negative? Zero?

3. Where does tan \( x \) have vertical asymptotes?

4. Does \( \lim_{x \to \frac{\pi}{2}} \tan x \) exist? Why or why not?

Trig Identities

Questions

1. On the unit circle below, label the coordinate of the point marked on the circle in terms of \( A \):

2. Use Pythagoras to derive an identity relating the lengths of the sides of the right-angled triangle in the diagram to the length of its hypotenuse:
The identity you just wrote down above is the most basic trigonometric identity, from which many others can be derived. Before we do so, we’ll introduce three additional trig functions:

- \( \sec \theta = \frac{1}{\cos \theta} = \frac{\text{hyp}}{\text{adj}} \) - The secant function;
- \( \csc \theta = \frac{1}{\sin \theta} = \frac{\text{hyp}}{\text{opp}} \) - The cosecant function;
- \( \cot \theta = \frac{1}{\tan \theta} = \frac{\text{adj}}{\text{opp}} = \frac{\cos \theta}{\sin \theta} \) - The cotangent function.

**Question**  Using the identity you derived above, show that the following identities are true:

- \( 1 + \cot^2 x = \csc^2 x \)
- \( \tan^2 x + 1 = \sec^2 x \)

**Angle Addition Formulas**

We will not prove the following, though they are true and will be very useful:

- \( \sin(A + B) = \sin A \cos B + \cos A \sin B \)
- \( \sin(A - B) = \sin A \cos B - \cos A \sin B \)
- \( \cos(A + B) = \cos A \cos B - \sin A \sin B \)
- \( \cos(A - B) = \cos A \cos B + \sin A \sin B \)

**Questions**

1. Use the formulas above to show that \( \sin(2A) = 2 \sin A \cos A \).

2. Show that \( \cos(\frac{\pi}{2} - x) = \sin(x) \).

You will see more of these on the homework...
An Important Limit

Using Wolfram Alpha, draw the graph of $f(x) = \frac{\sin x}{x}$ below:

Questions

1. Is $f(x)$ defined at $x = 0$? Why or why not?

2. By plugging in numbers near 0, complete the following:

$$\lim_{x \to 0} \frac{\sin x}{x} = \underline{\phantom{0000}}.$$

Question  Using Wolfram Alpha, draw the graph of $g(x) = \frac{\cos x - 1}{x}$ below:

Questions

1. Is $g(x)$ defined at $x = 0$? Why or why not?

2. By plugging in numbers near 0, complete the following:

$$\lim_{x \to 0} \frac{\cos x - 1}{x} = \underline{\phantom{0000}}.$$

Again, we will not prove that the limit you wrote down above is true, but we will use them when doing calculus with trig functions.
Extra Homework Questions

1. Use the formula for \( \sin(A + B) \) to find a formula for \( \sin(2A) \). (Note \( 2A = A + A \))

2. Graph \( y = \sin 2x \) and \( y = 2 \sin x \cos x \) together to see that they are the same.

3. Use the addition formulas to show that \( \cos \left( \frac{\pi}{2} - x \right) = \sin x \). Check by graphing the two functions together.

4. Use the formula for \( \cos(A + B) \) to show that \( \cos(2x) = \cos^2(x) - \sin^2(x) \). Then show that \( \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x \).

5. Use the formulas for \( \sin(A + B) \) and \( \cos(A + B) \) to complete the following derivation of the identity \( \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \): 

\[
\tan(A + B) = \frac{\sin(A + B)}{\cos(A + B)} = \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B} = \frac{\frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B}}{1 - \frac{\sin A \sin B}{\cos A \cos B}} = \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B}.
\]

6. Compute the following limits (you may not use L'Hôpital's rule here):

(a) \( \lim_{x \to 0} \frac{\sin(2x)}{x} \) (Hint: use your answer to Question 1)

(b) \( \lim_{x \to \infty} \frac{\sin(2x)}{x} \)

(c) \( \lim_{x \to 0} \frac{\sin^2 x}{x} \)

(d) \( \lim_{x \to 0} \frac{x}{\cos x} \)

(e) \( \lim_{x \to 0} \frac{x}{\sin x} \)

(f) \( \lim_{x \to 0} x \cot x \)

7. Prove the following identities:

(a) \( \csc^2 x - \cot^2 x = 1 \)

(b) \( \frac{\sin^2 x}{\cos x} + \cos x = \sec x \)

(c) \( \cos \theta \tan \theta \csc \theta = 1 \)