Review

\[ \int_a^b f(x) \, dx = \lim_{n \to \infty} (RHS)_n = \lim_{n \to \infty} (LHS)_n \]

where \( \Delta x = \) ____________.

The Fundamental Theorem of Calculus: If \( f(x) \) is continuous on \([a, b]\), and if _____ is an ______________ of \( f(x) \), then

\[ \int_a^b f(x) \, dx = \] ____________.

The Midpoint Sum

The midpoint sum with \( n \) subintervals for a function \( f(x) \) on the interval \([a, b]\) is given by

\[ (MPS)_n = \sum_{k=1}^{n} f \left( a + k\Delta x - \frac{1}{2}\Delta x \right) \Delta x. \]

Question Explain in words why the expression inside \( f \) in the sum results in the midpoint sum.

Question Draw and then calculate \((LHS)_2\), \((RHS)_2\) and \((MPS)_2\) for the following integrals:

1. \( \int_0^4 x^2 - 1 \, dx \)

\[ (LHS)_2 = \] _____  \( (RHS)_2 = \) _____  \( \frac{(LHS)_2 + (RHS)_2}{2} = \) _____  \( (MPS)_2 = \) _____

2. \( \int_0^4 \sqrt{x} - 2 \, dx \)

\[ (LHS)_2 = \] _____  \( (RHS)_2 = \) _____  \( \frac{(LHS)_2 + (RHS)_2}{2} = \) _____  \( (MPS)_2 = \) _____
3. Use the FTC to calculate the above integrals exactly.

4. When does it seem the MPS and the average of the LHS and RHS are over/underestimates of the true value of an integral?

Let’s demonstrate why your answer to the last question is true. To do this, we must recast these two Riemann sums as sum of areas of \textit{trapezoids}:

\begin{align*}
\text{Conclusion} \\
\bullet & \text{ If } f(x) \text{ is } \underline{\hphantom{123456}}, \text{ then } (MPS)_n \text{ is an underestimate for } \int_a^b f(x) \, dx \text{ and } \frac{(LHS)_2 + (RHS)_2}{2} \\
& \text{ is an overestimate for it.} \\
\bullet & \text{ If } f(x) \text{ is } \underline{\hphantom{123456}}, \text{ then } (MPS)_n \text{ is an overestimate for } \int_a^b f(x) \, dx \text{ and } \frac{(LHS)_2 + (RHS)_2}{2} \\
& \text{ is an underestimate for it.}
\end{align*}

Use this worksheet to complete Part V of the Riemann Sums lab.