Checking Solutions and Reading Equations

Just as solving an *algebraic* equation such as \( x^2 + 2x - 7 = x^3 \) involves finding a *number* \( x \) that makes the two sides of the equation equal, solving a *differential* equation such as

\[
\frac{dy}{dt} = y(t)
\]

involves finding a *function* \( y(t) \) that makes the two sides of the equation equal.

While the algebraic equation \( x^4 + x^3 = x^2 + 1 \) is quite hard to solve in general, *checking* whether a given number is a solution is easy: just plug in.

**Question**  Check that \( x = 1 \) is a solution to the above equation.

We will develop techniques to solve various types of differential equations later in the semester, but for now, let’s just try reading...

**Questions**

1. Consider the differential equation \( y'(t) = y(t) \). Write down what this differential equation says in your own words:

(a) What function \( y(t) \) satisfies the sentence you just wrote down? Plug in to both sides of the equation to check it.

(b) Is the function \( y(t) = 2 + e^t \) also a solution?

(c) Can you figure out another function that solves the equation? Check it!

(d) Can you write down the general solution to this equation?
2. Write down what the following differential equation says in words: \( \frac{dy}{dt} = 2y(t) \).

(a) Can you figure out a function that solves this equation? Can you find the general solution?

(b) Below, draw the graphs of your solution with initial conditions \( y(0) = 0 \), \( y(0) = 1 \) and \( y(0) = -1 \). Put all the graphs on the same set of axes.

Order of Differential Equations - Families of Solutions

Definition  The order of a differential equation is the highest derivative that appears in it.

Questions  Solve the differential equation \( \frac{d^2y}{dt^2} = 12t^2 \) by antidifferentiation. How many arbitrary constants does your solution contain?

In general, an \( n \)'th order differential equation will have \( n \) arbitrary constants in its solution.

Questions  

1. What is the order of the differential equation

\[
\frac{d^2y}{dt^2} + 9y(t) = 0
\]
2. Check that \( y(t) = \sin(3t) \) is a solution for this equation.

3. Any guesses at function that doesn’t involve sin that solves this equation? Check your guess.

4. Take a guess at the general solution. Make sure you have the appropriate number of arbitrary constants, and check your answer! (Hint: look at where the constant went when we solved \( \frac{dy}{dt} = y(t) \) and \( \frac{dy}{dt} = 2y(t) \).)

5. Find the solution to the initial value problem: \( \frac{d^2y}{dt^2} + 9y(t) = 0 \) with \( y(0) = 1 \) and \( y'(0) = 1 \).

6. Find the solution to the initial value problem: \( \frac{d^2y}{dt^2} + 9y(t) = 0 \) with \( y(0) = -2 \) and \( y'(0) = 3 \).

7. Graph the previous two answers below.
What we can and can’t do (for now)

• The only differential equations we can currently solve analytically— that is, without (much) guessing—are those of the form

\[ \frac{dy}{dt} = f(t), \]

such as \( \frac{dy}{dt} = t^2 \). We solve these by finding an \[ \underline{\text{__________}} \] of \( f(t) \).

• We cannot (for now) solve equations like

\[ \frac{dy}{dt} = y^2. \]

We will do (some of) these later in the semester.

• We cannot solve second (or higher) order differential equations other than \( \frac{d^2y}{dt^2} = f(t) \). We will leave other second order equations for a later course.

• We can also check whether a given function solves a particular differential equation.