Stuff We Already Know

- Given a right-angled triangle with angle $x$, write down the definitions of the following:

\[
\begin{align*}
\sin x &= \quad \quad \quad \quad , \\
\cos x &= \quad \quad \quad \quad \\
\tan x &= \quad \quad \quad \quad , \\
\sec x &= \quad \quad \quad \quad \\
\csc x &= \quad \quad \quad \quad , \\
\cot x &= \quad \quad \quad \quad
\end{align*}
\]

- Write down the following limits:

\[
\lim_{x \to 0} \frac{\sin x}{x} = \quad , \quad \lim_{x \to 0} \frac{\cos x - 1}{x} = \quad.
\]

- Write down the definition of the derivative for a function $f(x)$:

Derivatives of sin and cos

Questions

1. Graph the function, $g(x) = \frac{\sin(x + 0.001) - \sin(x)}{0.001}$ on the axes below with domain $[-2\pi, 2\pi]$. What does this approximate?

2. What other function that we know does this graph look like? Graph it on top of your graph of $g(x)$.

3. Write down a formula (using a limit) that would give the derivative of $\sin x$:

\[
\frac{d}{dx} (\sin x) = \lim \quad \quad \quad \quad .
\]
4. Use the fact that $\sin(A + B) = \sin A \cos B + \cos A \sin B$ to rewrite the derivative of $\sin(x)$ as the sum of two different limits.

5. Use your two limits to find what $\frac{d}{dx} \sin x$ should be.

**Question** Use the same sorts of tricks to find $\frac{d}{dx} \cos x$:
(Note: $\cos(A + B) = \cos A \cos B - \sin A \sin B$.)
Other Trig Derivatives

Questions

1. Compute \( \frac{d}{dx} \tan x \). (Hint: Quotient Rule)

2. Compute \( \frac{d}{dx} \sec x \), \( \frac{d}{dx} \csc x \), and \( \frac{d}{dx} \cot x \). (Hint: for example, write \( \sec x = \frac{1}{\cos x} = (\cos x)^{-1} \) and use the chain rule...)

The derivatives we found above are important. For future reference, write them below:

\[
\frac{d}{dx} \sin x = \quad \frac{d}{dx} \cos x = \\
\frac{d}{dx} \csc x = \quad \frac{d}{dx} \sec x = \\
\frac{d}{dx} \tan x = \quad \frac{d}{dx} \cot x =
\]
Questions

1. Find the equation of the tangent line to the graph of $f(x) = \tan x$ at $x = 1$.

2. A particle is moving along a straight line. Its position from its origin (in meters) at time $t$ seconds is given by $s(t) = \sin^2 t$. Find its velocity and acceleration at time $t = 2$ seconds. At what times is the particle at rest? (Hint: The identity $\sin 2x = 2 \sin x \cos x$ may be useful.)

3. (a) Find the linear approximation to the curve $g(x) = \sec x$ at $x = \frac{5\pi}{6}$.

(b) Use your answer above to estimate $\sec \frac{11\pi}{12}$. 