A Key Tool!

The chain rule:

$$\frac{d}{dx}(f(g(x))) = \underline{\hspace{3cm}}.$$

Implicit Differentiation

Questions

1. Consider the equation $xy + y = e^x$. This defines y as function of x. Find $y(x)$ and use it to find $\frac{dy}{dx}$.

2. Now, consider the equation $x^2 + y^2 = 4$.

   (a) What curve does this equation describe? Draw it!

   (b) The equation $x^2 + y^2 = 4$ does not define y as a function of x. Why not?

   (c) However, "near the point $(0, 2)$", y is function of a x. What is this function? What is the derivative of this function at $x = 1$?

   (d) Do the same for the point $(0, -2)$ (and $x = -1$).

   (e) Notice that in both the above cases, we can write $\frac{dy}{dx} = \underline{\hspace{1.5cm}}$.

(f) Even though $x^2 + y^2 = 4$ does not define y as a function of x, it is still close to a function that the derivative makes sense, and the usual rules apply. Let's pretend y is a function of x, differentiate both sides, and solve for $\frac{dy}{dx}$:

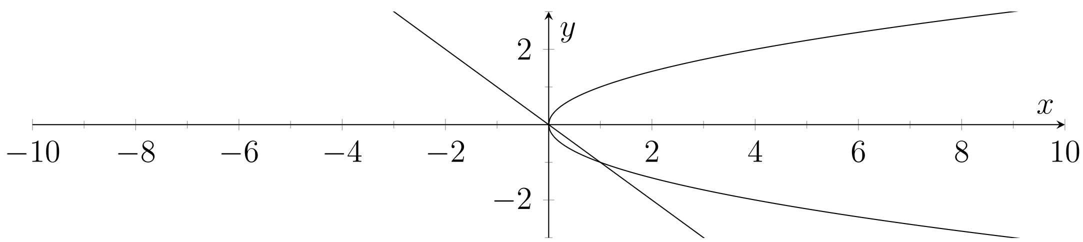
(g) Notice that $\frac{dy}{dx}$ depends on both x and y. For "normal" functions, the derivative should only depend on the independent variable. Why isn't this true for our equation?

(h) Are there any points where $\frac{dy}{dx}$ is not defined? What happens to the tangent line to the curve at these points?

3. Let's revisit $xy + y = e^x$. What is $\frac{dy}{dx}$? Check that it's the same as what you got previously!

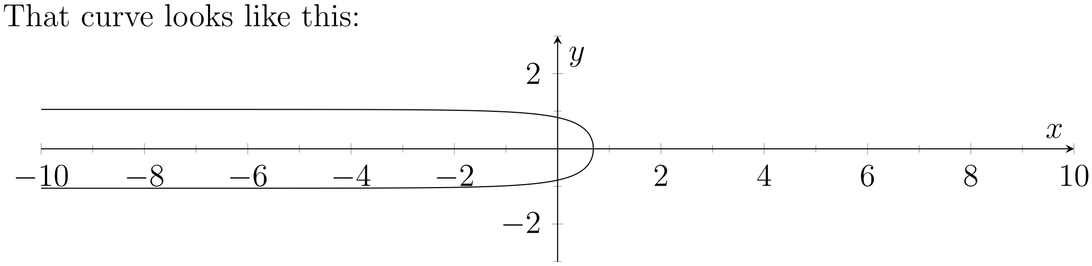
4. Find the tangent line to $x^2 + xy - y^3 = xy^2$ at the point $(1, 1)$.

Here's a picture of this curve. Odd, eh? What happens to $\frac{dy}{dx}$ at $(0, 0)$ and $(1, -1)$? More on this in lab...



5. (a) Find $\frac{dy}{dx}$ if $e^x + e^{y^2} = 3$.

That curve looks like this:



(b) As x approaches $-\infty$, what does $\frac{dy}{dx}$ approach? Make sure you see how this matches up with the graph!

(c) At the point $(\ln(2), 0)$, what is $\frac{dy}{dx}$? What direction is the tangent line at that point?

6. Where does the curve $y^3 - xy = 1$ have

   - a vertical tangent? (Hint: see question 5c.)

   - a horizontal tangent? (Hint: if the tangent line is horizontal, what must $\frac{dy}{dx}$ be?)

Here's a picture of this curve. Check that your two answers above make sense in the context of the picture!