Back to Calculus...

- Given a function $f(x)$, its *derivative function* is given by

$$f'(x) = \rule{4cm}{0.4pt}.$$

- To approximate the derivative of a function at a point $x = a$, we plug in small values of ___ into the *difference quotient*

$$f'(a) \approx \rule{4cm}{0.4pt}.$$

- The quantity A is directly proportional to the quantity B if $A =$ ______.

Playing with Derivatives

Let $f(x) = 2^x$ and $g(x) = 3^x$. Using a spreadsheet to approximate the derivatives, fill out the following table.

x	$f(x)$	$f'(x)$	$\frac{f'(x)}{f(x)}$	$g(x)$	$g'(x)$	$\frac{g'(x)}{g(x)}$
-4						
-3						
-2						
-1						
0						
1						
2						
3						
4						

Sample calculation for $x = -4$:

Questions

1. It seems that $f(x)$ is ____________ ____________ to its derivative. Mathematically, it seems that $f'(x) = k$___. Use the limit definition of the derivative to show that this is indeed the case. What can you say about k?

2. If $h(x) = a^x$, show a similar relationship between $h'(x)$ and $h(x)$.

3. Fill in the following: if $h(x) = a^x$, then

$$h'(x) = k\_\_\_, \text{ where } k = \lim \_\_\_\_\_\_\_\_\_\_.$$

4. Suppose we could find a number a such that the constant of proportionality becomes 1? What would $\frac{d}{dx}a^x$ be in that case? Using guess and check, estimate this a to two decimal places (take $h = 0.001$).

Why e is Important in Calculus

Such an a satisfies $\frac{a^h - 1}{h} \approx 1$ for small h. We can rewrite this as

$$a \approx$$

In the limit as $h \to 0$ we get a number we've encountered before: We define

$$e = \lim_{h \to 0} (h+1)^{\frac{1}{h}}.$$

This is the same e we saw before, when we did compound interest and natural logs. Our previous results give us that

Derivative of e^x

$$\frac{d}{dx}e^x = e^x$$

Now, back to the general exponential function b^x. By the chain rule, $\frac{d}{dx}e^{kx} =$ ____________. Then, we note that

$$b^x = \qquad\qquad = $$

so, using the chain rule result above:

$$\frac{d}{dx}b^x = \qquad\qquad = \qquad\qquad = $$

Derivatives of Exponential Functions

$$\frac{d}{dx}b^x = $$

Questions

5. Find the derivatives of the following functions:
   (a) $f(x) = 8x + 8^x$
   (b) $g(x) = e^2 - 2^x$
   (c) $h(x) = e^\pi + e^x + x^e$
6. Find the tangent line to x^2e^x at $x = 1$.
7. Find the tangent line to $e^{(x+1)^3}$ at $x = 1$ (Hint: see the triple chain rule from the last worksheet).