Past Differentiation...

The differentiation rules we’ve learned so far:

- Power Rule:

- Sum Rule:

- Difference Rule:

- Product Rule:

- Quotient Rule:

Motivation!

Questions

1. With the tools we have, how do we find \( \frac{d}{dx}(x^3 + 1)^3 \)? Is there any easier way?

2. (a) (From a previous worksheet): Let \( L(E) \) be the life expectancy (years) of a person with \( E \) years of education. Let \( E(i) \) be the average number of years of education obtained by people whose parents have an average annual income of \( i \) dollars. Describe the function \( L(E(i)) \) in practical terms. Include units in your answer.

(b) What are the units of \( \frac{dL}{dE}, \frac{dE}{di}, \text{ and } \frac{dL}{di} \)?
The Chain Rule

More general question: How do we differentiate the composition of two functions (functions inside of functions)?

<table>
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<th>The Chain Rule</th>
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<td>If $f(x)$ and $g(x)$ are differentiable, then</td>
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<td>$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$</td>
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If we let $z = g(x)$ and $y = f(z)$, then $y = f(g(x))$ and we can write the chain rule in the following way:

$$ \frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx} $$

Questions

3. Differentiate the functions below:

   (a) $(x^3 + 1)^3$ (How does this compare to our answer above?)

   (b) $(x^2 + 1)^{100}$

   (c) $(8x^2 + 2)^7$.

   (d) $\left(\frac{1}{x^2} + x + 1\right)^3$. 
(e) \( \left( \frac{(2x+3)^2}{(x+1)^3} \right)^{\frac{3}{2}} \).

4. A new proof of the quotient rule using the chain and product rules:

5. Find the equation of the tangent line to \( f(x) = (\frac{2}{x} - 1)^3 \) at the point where \( x = 3 \).

6. The Triple Chain Rule: What about the composition of three functions, \( f(g(h(x))) \)? How would we differentiate that?
Homework Problems

You are driving north from Durham. The temperature gets colder and colder farther and farther north of Durham. The function \( F(x) \) gives the temperature, in degrees Fahrenheit, \( x \) miles north of Durham. The function \( s(t) \) gives your distance north of Durham, in miles, after you have been driving for \( t \) hours.

1. Explain the meaning of the function \( f(t) = F(s(t)) \).

2. What are the units of \( f'(t) \)?

3. Explain the meaning of each of the following (give units):
   (a) \( s(3) = 160 \)
   (b) \( s'(3) = 80 \)
   (c) \( F(160) = 75 \)
   (d) \( F'(160) = -0.05 \)

4. After 3 hours of driving, how fast is the temperature changing outside your car? Give units. Use your answers from above.

5. \( f'(3) = \) ___________________ (remember to use units)

6. Which of the following is the correct formula for \( f'(3) \)? Circle your answer:
   - \( f'(3) = F'(3) \cdot s'(3) \)
   - \( f'(3) = F'(s'(3)) \)
   - \( f'(3) = F'(s(3)) \cdot s'(3) \)