Past Differentiation...

The differentiation rules we've learned so far:

- Power Rule:
- Sum Rule:
- Difference Rule:
- Product Rule:
- Quotient Rule:

Motivation!

Questions

1. With the tools we have, how do we find $\frac{d}{dx}(x^3+1)^3$? Is there any easier way?

2. (a) (From a previous worksheet): Let $L(E)$ be the life expectancy (years) of a person with E years of education. Let $E(i)$ be the average number of years of education obtained by people whose parents have an average annual income of i dollars. Describe the function $L(E(i))$ in practical terms. Include units in your answer.

   (b) What are the units of $\frac{dL}{dE}$, $\frac{dE}{di}$, and $\frac{dL}{di}$?

The Chain Rule

More general question: How do we differentiate the composition of two functions (functions inside of functions)?

The Chain Rule
If $f(x)$ and $g(x)$ are differentiable, then

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x)) \cdot g'(x)$$

If we let $z = g(x)$ and $y = f(z)$, then $y = f(g(x))$ and we can write the chain rule in the following way:

$$\frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx}$$

Questions

3. Differentiate the functions below:

   (a) $(x^3 + 1)^3$ (How does this compare to our answer above?)

   (b) $(x^2 + 1)^{100}$

   (c) $(8x^2 + 2)^7$.

   (d) $\left(\frac{1}{x^2} + x + 1\right)^3$.

(e) $\left(\frac{(2x+3)^2}{(x+1)^3}\right)^{\frac{3}{2}}$.

4. A new proof of the quotient rule using the chain and product rules:

5. Find the equation of the tangent line to $f(x) = (\frac{2}{x} - 1)^3$ at the point where $x = 3$.

6. The Triple Chain Rule: What about the composition of three functions, $f(g(h(x)))$? How would we differentiate that?

Homework Problems

You are driving north from Durham. The temperature gets colder and colder farther and farther north of Durham. The function $F(x)$ gives the temperature, in degrees Fahrenheit, x miles north of Durham. The function $s(t)$ gives your distance north of Durham, in miles, after you have been driving for t hours.

1. Explain the meaning of the function $f(t) = F(s(t))$.

2. What are the units of $f'(t)$

3. Explain the meaning of each of the following (give units):

   (a) $s(3) = 160$

   (b) $s'(3) = 80$

   (c) $F(160) = 75$

   (d) $F'(160) = -0.05$

4. After 3 hours of driving, how fast is the temperature changing outside your car? Give units. Use your answers from above.

5. $f'(3) =$ ____________ (remember to use units)

6. Which of the following is the correct formula for $f'(3)$? Circle your answer:

   - $f'(3) = F'(3) \cdot s'(3)$
   - $f'(3) = F'(s'(3))$
   - $f'(3) = F'(s(3)) \cdot s'(3)$