Review - Exponential Functions

- An exponential function is any function of the form

$$f(x) = \underline{\hspace{2cm}},$$

  where $a > 0$.

- P_0 is the ____________ ____________ of the function. In other words, P_0 is the ___-____________ of the graph of the function.
- If $a > 1$, then the function is ____________.
- If $0 < a < 1$, then the function is ____________.
- A bank account that pays interest rate r, compounded continuously, will yield $__________ if we leave $A in it for t years.

Logarithms

Question Is every exponential function invertible? How do you know?

On the last worksheet, you were asked to find the doubling time of a bank account with interest rate 5%, compounded continuously. This required you to solve the equation

$$2A = Ae^{0.05t}$$

for t (where A is the initial deposit). To do so, you plugged in various numbers for t until you got pretty close to the answer. This is a little unsatisfactory.

In other terms, if we define $P(t) = Ae^{0.05t}$, we wish to compute ____________.

If $f(x) = 10^x$, then we define the *logarithm base 10* of x to be $f^{-1}(x)$. In other words

$$\log_{10} x = c \Leftrightarrow 10^c = x.$$

Often, we leave off the 10 and just write $\log x$.

If $f(x) = e^x$, then we define the *logarithm base e* of x to be $f^{-1}(x)$. In other words

$$\log_e x = c \Leftrightarrow e^c = x.$$

For reasons we'll see next time, log base e is called the *natural logarithm* and is most often written $\log_e x = \ln x$.

Questions

1. (a) What are the range and domain of $\log x$?

   (b) On the same axes, draw the graphs of 10^x and $\log x$. (Hint: recall that the graph of the inverse of a function $f(x)$ is given by reflecting the graph of $f(x)$ in the line ________).

   (c) Why does it make no sense to find $\log(0)$? What about $\log(x)$ when x is negative?

   (d) Does $\log(x)$ have a vertical asymptote? Where?

   (e) Do all the previous questions for $\ln(x)$.

Properties of Logarithms

All the following properties can be deduced from the properties of exponents (eg $x^{a+b} = x^a x^b$):

$\log(AB) = \log A + \log B$	$\ln(AB) = \ln A + \ln B$
$\log\left(\frac{A}{B}\right) = \log A - \log B$	$\ln\left(\frac{A}{B}\right) = \ln A - \ln B$
$\log(A^p) = p\log(A)$	$\ln(A^p) = p\ln(A)$
$\log(10^x) = x$	$\ln(e^x) = x$
$10^{\log x} = x$	$e^{\ln x} = x$

Applications of Logs

Questions

2. Find the doubling time of a bank account that has 5% interest rate compounded continuously.

3. Suppose we start with 10 moles of radon-222, a radioactive element. After two days, we find that there are 6.943 moles of radon-222 remaining. Find the half-life of radon-222.

4. Find the inverses of the following functions:

   (a) $f(t) = 10(7)^t$

   (b) $g(t) = 2\ln(t) + 5$

5. Solve the following equations:

   (a) $7^x = 2$

   (b) $10^{2x+9} = 6 \cdot 7^{x-4}$

   (c) $9^{7-2x} + 6 = e$

6. Let $f(t) = 7(2^t)$. Write $f(t)$ in the form $P_0 e^{rt}$. (Hint: $e^{rt} = (e^r)^t$. If you can figure out P_0 and r, you're done...)

Note: As seen in the last question, it is possible to write any exponential function in the form $P(t) = P_0 e^{rt}$. This is far more common than $P(t) = P_0 a^t$, so we'll be using it from now on.

Homework Problems

1. (a) Can the log of a number be negative? Explain.

   (b) Can we take the log of a negative number? Explain.

2. State the domain and range of the function $y = \log(x)$.

3. Evaluate the following expressions <u>without</u> using a calculator.

   (a) $\log\left(\frac{1}{100}\right)$

   (b) $\log\left(200^2\right) - \log(40)$

   (c) $\log\left(\frac{1}{10,000}\right) + \log\sqrt{1,000}$

4. Let $x = \log A$ and $y = \log B$. Rewrite the following expressions in terms of x and y.

   (a) $\sqrt{\log(AB)}$

   (b) $\dfrac{\log B}{\log A}$

   (c) $\log\left(\sqrt{A}B^{-2}\right)$

   (d) $\log\left(\frac{A}{B}\right)$

5. If possible, use the properties of logarithms to find exact solutions of the the following equations for x.

   (a) $\log(1 - x) - \log(1 + x) = 2$

   (b) $\log(10x - 4) \cdot \log(16x^2) = 0$

   (c) $\frac{1}{5} \cdot 5^x - 25 = 100$

   (d) $\log(6x) - \log(2x - 1) = 2$