And Once Again, The Past...

1. The derivative of a function $f(x)$ is

2. A function is continuous at a point $x = a$ if

Differentiability

**Question**  If we zoom in far enough near any point of the graph of the function $f(x) = x^2$, the graph looks like a __________ __________

**Question**  What about $f(x) = |x|$ near 0? Do we get the same thing? What is going on?

A function $f$ is *differentiable* at a point $a$ if the ______________ of $f(x)$ exists and is finite at $x = a$. In other words, $f(x)$ is differentiable at $x = a$ if

$$
\lim_{{h \to 0}} \frac{f(a + h) - f(a)}{h}
$$

exists and is finite.

**Questions**

1. Show from the definition that the following functions are not differentiable at $x = 0$:
   
   (a) $f(x) = |x|

   (b) $f(x) = x^{\frac{1}{3}}$
(c) \( f(x) = \begin{cases} 
-1 & \text{if } x > 0 \\
1 & \text{if } x \leq 0 
\end{cases} \)

2. What can cause a function not to be differentiable at a point \( x = a \)? Draw an example of each.

- If the function

- If the function

- If the function

**Continuity and Differentiability**

**Theorem** If \( f(x) \) is \[ \text{________________________} \] at a point \( x = a \), then \( f(x) \) is \[ \text{________________________} \] at \( x = a \).

**Questions**

1. Fill in the following blanks with the words “continuous” and “differentiable”:

- If \( f(x) \) is not \[ \text{________________________} \] at \( x = a \), it is not \[ \text{________________________} \] at \( x = a \).

- If \( f(x) \) is \[ \text{________________________} \] at \( x = a \), it is \[ \text{________________________} \] at \( x = a \).

- It is possible for \( f(x) \) to be \[ \text{________________________} \] at \( x = a \), but not \[ \text{________________________} \] at \( x = a \).
2. (a) Find $f'(0)$ or explain why it does not exist if

$$f(x) = \begin{cases} 
4 - x^2 & \text{if } x > 0 \\
 x^2 - 4 & \text{if } x \leq 0
\end{cases}.$$

(b) Find $f'(0)$ or explain why it does not exist if

$$f(x) = \begin{cases} 
4 - x^2 & \text{if } x > 0 \\
 x^2 + 4 & \text{if } x \leq 0
\end{cases}.$$

3. Find $f'(0)$ or explain why it does not exist if $f(x) = (x + |x|)^2 + 1$.

(Hint: Write $|x|$ as a piecewise function!)
4. Find $a$ and $b$ such that the following function is differentiable everywhere:

$$f(x) = \begin{cases} 
ax^3 & \text{if } x \leq 2 \\
x^2 + b & \text{if } x > 2 
\end{cases}.$$  

(Hint: First make $f(x)$ continuous, then differentiate...)