And Once Again, The Past...

I The derivative of a function $f(x)$ is

II A function is continuous at a point $x = a$ if

Differentiability

Question If we zoom in far enough near any point of the graph of the function $f(x) = x^2$, the graph looks like a _____ _____

Question What about $f(x) = |x|$ near 0? Do we get the same thing? What is going on?

A function f is *differentiable* at a point a if the _____ of $f(x)$ exists and is finite at $x = a$. In other words, $f(x)$ is differentiable at $x = a$ if

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

exists and is finite.

Questions

1. Show from the definition that the following functions are not differentiable at $x = 0$:

   (a) $f(x) = |x|$

   (b) $f(x) = x^{\frac{1}{3}}$

(c) $f(x) = \begin{cases} -1 & \text{if } x > 0 \\ 1 & \text{if } x \leq 0 \end{cases}$

2. What can cause a function not to be differentiable at a point $x = a$? Draw an example of each.

   - If the function

   - If the function

   - If the function

Continuity and Differentiability

Theorem If $f(x)$ is _____ at a point $x = a$, then $f(x)$ is _____ at $x = a$.

Questions

3. Fill in the following blanks with the words "continuous" and "differentiable":

   - If $f(x)$ is not _____ at $x = a$, it is not _____ at $x = a$.
   - If $f(x)$ is _____ at $x = a$, it is _____ at $x = a$.
   - It is possible for $f(x)$ to be _____ at $x = a$, but not _____ at $x = a$.

4. (a) Find $f'(0)$ or explain why it does not exist if

$$f(x) = \begin{cases} 4 - x^2 & \text{if } x > 0 \\ x^2 - 4 & \text{if } x \leq 0 \end{cases}.$$

(b) Find $f'(0)$ or explain why it does not exist if

$$f(x) = \begin{cases} 4 - x^2 & \text{if } x > 0 \\ x^2 + 4 & \text{if } x \leq 0 \end{cases}.$$

5. Find $f'(0)$ or explain why it does not exist if $f(x) = (x + |x|)^2 + 1$.
(Hint: Write $|x|$ as a piecewise function!)

6. Find a and b such that the following function is differentiable everywhere:

$$f(x) = \begin{cases} ax^3 & \text{if } x \leq 2 \\ x^2 + b & \text{if } x > 2 \end{cases}.$$

(Hint: First make $f(x)$ continuous, then differentiate...)