From Last Time

1. If $f' > 0$ on a interval, then f is ______________________________ on that interval.
2. If $f' < 0$ on a interval, then f is ______________________________ on that interval.

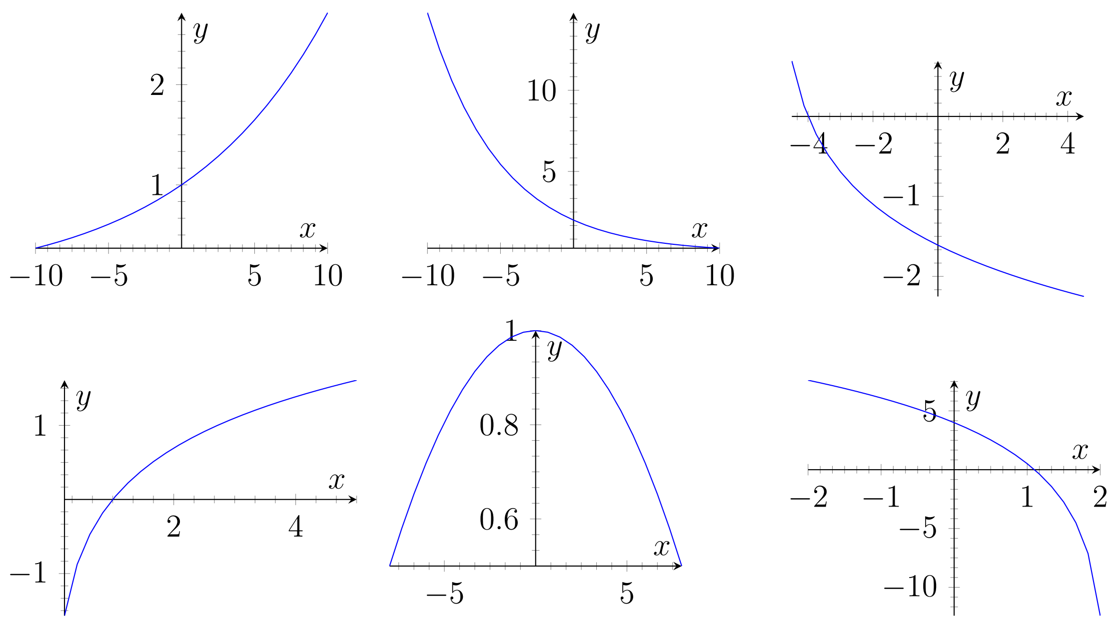
The Second Derivative

The **second derivative** of a function f is the derivative of the derivative function. Notationally, this means

$$f''(x) = \lim_{h \to 0} \frac{f'(x+h) - f'(x)}{h}$$

Questions

1. For each of the following functions, draw tangent lines to the curve on the left-hand side of the graph, in the middle, and on the right-hand side of the graph. What does the trend of your slopes tell you about the sign of the second derivative? Why?



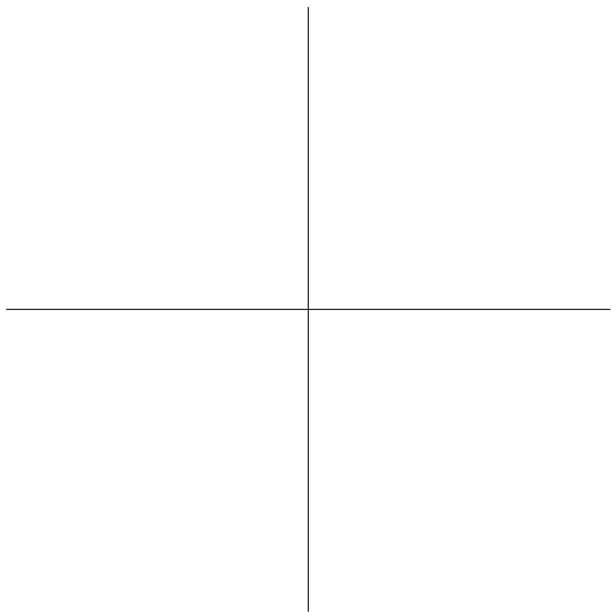
2. Fill in the blanks:

   (a) If $f'' > 0$ on an interval, then f' is ______________________________, so the graph of f is ______________________________.

   (b) If $f'' < 0$ on an interval, then f' is ______________________________, so the graph of f is ______________________________.

> **Beware!** The converses of the above statements aren't true! See the next question!

3. Graph the functions $f(x) = x^4$ and $g(x) = -x^4$ below. What can you say about each of them in terms of concavity?



Note that $f''(0) = g''(0) = 0$. (See Lab - Specifically, the Power Rule)

4. So, for a general function f:

   - If the graph of f is concave up, then ________________________.
   - If the graph of f is concave down, then ________________________.

> Let $s(t)$ be the position (or distance) function for an object at time t. Remember that the velocity, $v(t) = s'(t)$ was
>
> $$v(t) = s'(t) = \lim_{h \to 0} \frac{s(t+h) - s(t)}{h}$$
>
> We define the **acceleration** at time t, $a(t)$, to be the rate of change in the velocity:
>
> $$a(t) = v'(t) = s''(t)$$

Graphs with Specified Properties

Questions For the following five questions, draw the graph of a function f having the desired properties or explain why such a function does not exist.

5. $f'(x) > 0$ for all x, $f''(x) < 0$ for $x < 0$, $f''(x) > 0$ for $x > 0$ and $\lim_{x\to\infty} f(x) = 5$.

6. f has exactly two zeroes, f' has exactly two zeroes, and $f''(x) < 0$ when $0 < x < 4$.

7. f has exactly one point at which concavity changes and has three zeroes.

8. f has exactly one point at which concavity changes and has two zeroes.

9. f has exactly one point at which concavity changes and has no zeroes.

10. For the graphs you drew above, label all points at which concavity changes. Draw the tangent lines to the curves at those points. What is special about the tangent line at a point at which concavity changes?