Preliminaries: Review

- If $f' > 0$ on a interval, then f is _____ on that interval.
- If $f' < 0$ on a interval, then f is _____ on that interval.

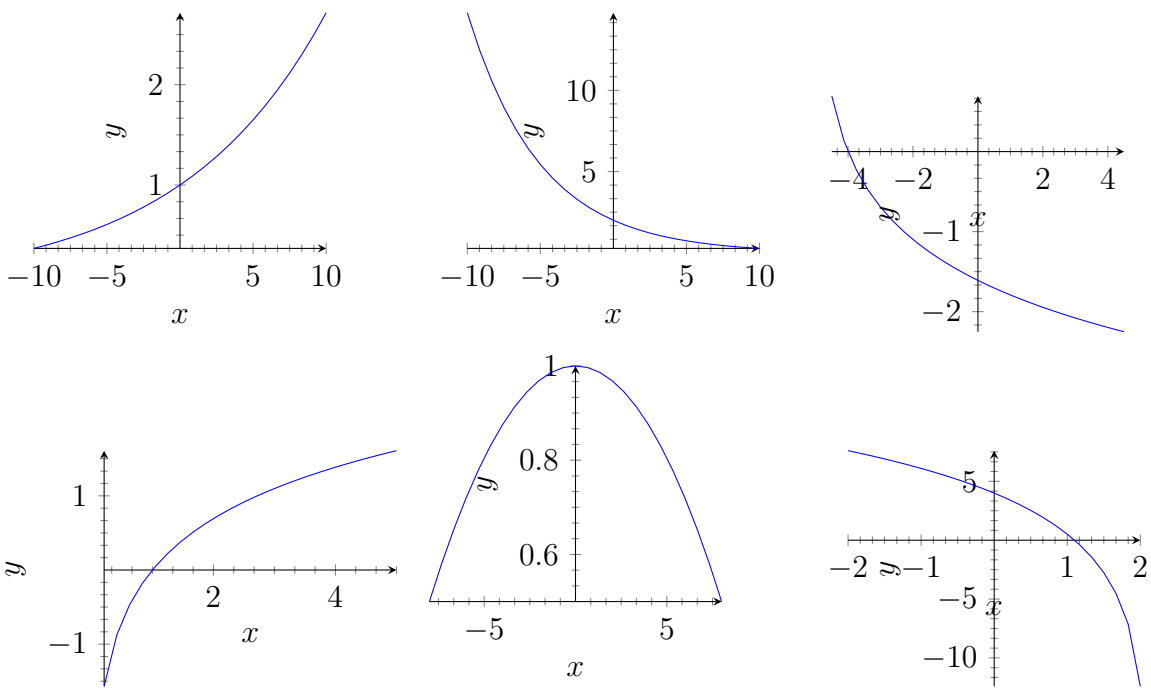
The Second Derivative

The **second derivative** of a function f is the derivative of the derivative function. Notationally, this means

$$f''(x) = \lim_{h \to 0} \frac{f'(x+h) - f'(x)}{h}$$

Since the first derivative of a function, $f'(x)$ tells you what the slope of the function *is*, its derivative, $f''(x)$ tells you what the slopes are *doing*.

For each of the following functions, draw tangent lines to the curve on the left-hand side of the graph, in the middle, and on the right-hand side of the graph.



Fill in the blanks:

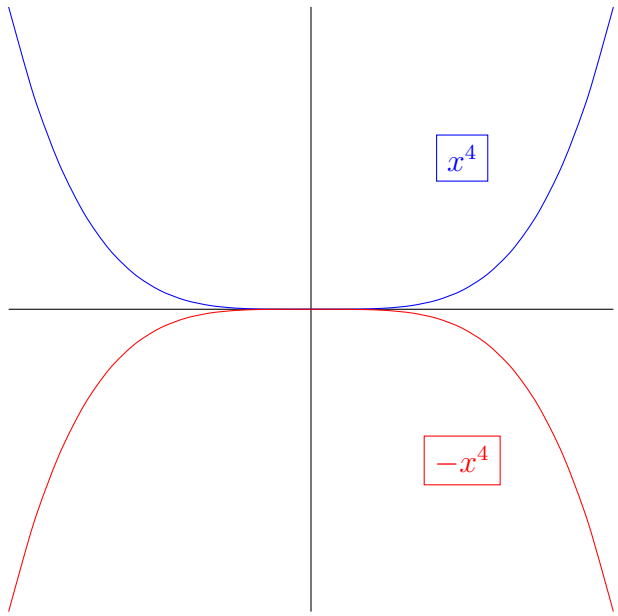
- In the first row of graphs, as x increases, the slope of the tangent lines _____.
- In the second row of graphs, as x increases, the slope of the tangent lines _____.

What does the trend of your slopes tell you about the sign of the second derivative? Fill in the blanks:

- If $f'' > 0$ on an interval, then f' is _____, so the graph of f is _____.
- If $f'' > 0$ on an interval, then f' is _____, so the graph of f is _____.

Beware! The converses of the above statements aren't true! See below!

The graphs of $f(x) = x^4$ and $g(x) = -x^4$ are shown below. What can you say about the concavity of each of them?



As we will see in lab, $f''(x) = 12x^2$ and $g''(x) = -12x^2$. Note that $f''(0) = g''(0) = 0$! So it is not always true that if $f(x)$ is concave up, then $f''(x) > 0$. But it is true that:

- If the graph of f is concave up, then _____.
- If the graph of f is concave down, then _____.

Let $s(t)$ be the position (or distance) function for an object at time t. Remember that the velocity, $v(t) = s'(t)$ was

$$v(t) = s'(t) = \lim_{h \to 0} \frac{s(t+h) - s(t)}{h}$$

We define the **acceleration** at time t, $a(t)$, to be the rate of change in the velocity:

$$a(t) = v'(t) = s''(t)$$

Graphs with Specified Properties

Questions For the following five questions, draw the graph of a function f having the desired properties or explain why such a function does not exist.

1. $f'(x) > 0$ for all x, $f''(x) < 0$ for $x < 0$, $f''(x) > 0$ for $x > 0$ and $\lim_{x\to\infty} f(x) = 5$.

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2. f has exactly two zeroes, f' has exactly two zeroes, and $f''(x) < 0$ when $0 < x < 4$.

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3. f has exactly one point at which concavity changes and has three zeroes.

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4. f has exactly one point at which concavity changes and has two zeroes.

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5. f has exactly one point at which concavity changes and has no zeroes.

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Inflection Points

Look back at the graphs on the first page of worksheet and fill in the blanks:

- If a graph is concave up, its tangent lines lie _____ the graph.
- If a graph is concave down, its tangent lines lie _____ the graph.

An ***inflection point*** is a point where a graph changes from concave up to concave down (or vice versa). Therefore, the tangent line at an inflection point lies _____ the graph on one side of an inflection point and _____ the graph on the other side. That is, the tangent line at an IP *crosses the graph.*

6. For each of your graphs in the questions above, draw tangent lines at the inflection points. They should cross the graphs!

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