In Previous Classes....

I The *derivative* of a function at a point $x = a$ is

$$\lim \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}.$$

II $\lim_{x \to c} f(x) = L$ means we get ____________ as close to ____________ as we want by taking x ____.

The Derivative Function

When we evaluated the derivative of $f(x) = x^2$ at $a = 2$, we carried out the following calculation:

$$\begin{aligned} f'(2) &= \lim_{h \to 0} \frac{(2+h)^2 - 2^2}{h} \\ &= \lim_{h \to 0} \frac{4 + 4h + h^2 - 4}{h} \\ &= \lim_{h \to 0} \frac{4h + h^2}{h} \\ &= \lim_{h \to 0} 4 + h = 4 \end{aligned}$$

To evaluate the derivative of the same function at $a = 4$, say, we carry out an almost identical calculation. This is pretty pointless. Instead, for a given function $f(x)$, we calculate the *derivative function*, $f'(x)$:

Definition: Given a function $f(x)$, its *derivative function* is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

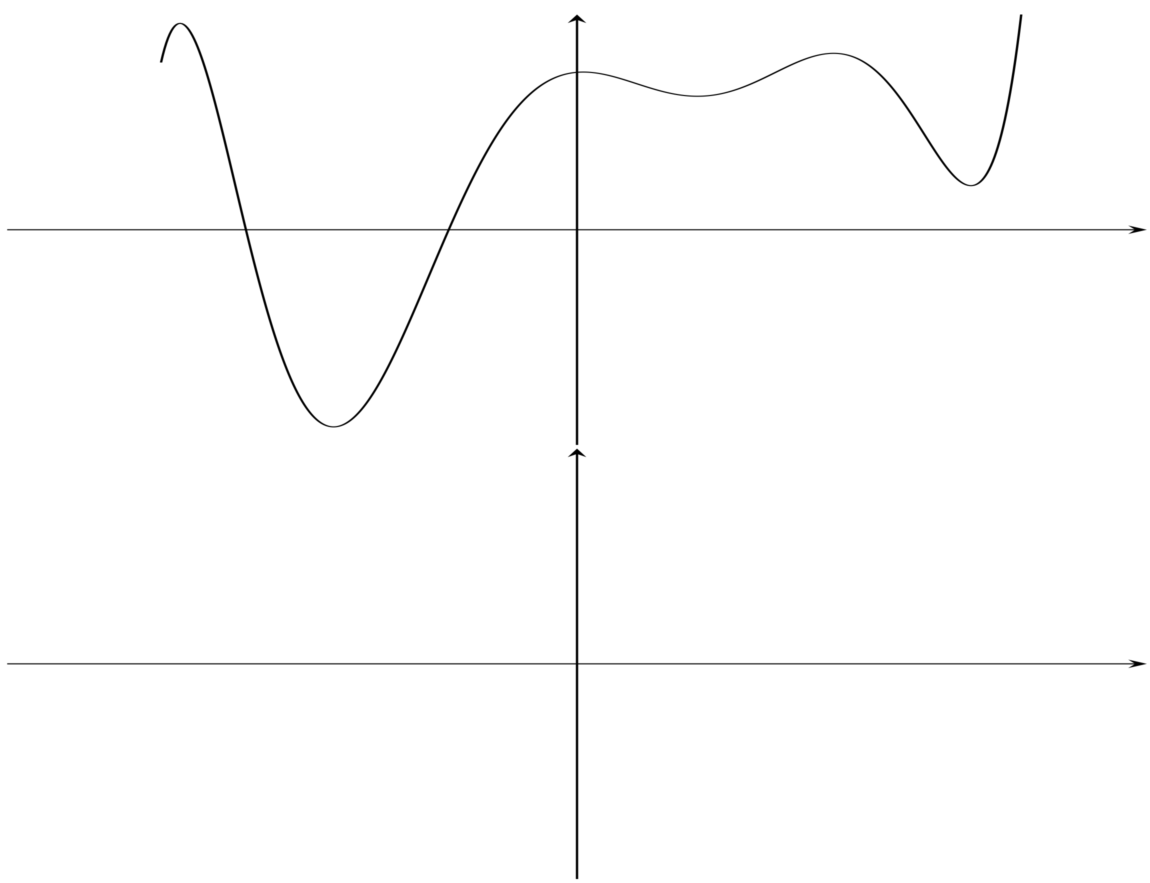
Note: The following are equivalent ways to denote the derivative of $y = f(x)$:

Question Fill in the blank: If $f(x)$ is a function, then $f'(a)$ gives the ____________ of $f(x)$ at the point $x = a$.

Playing with the Derivative Function

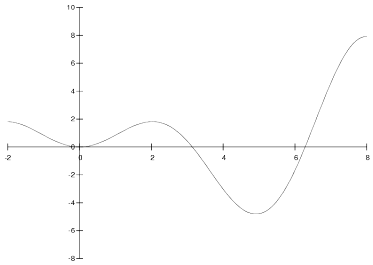
Questions

1. Examine the graph of a function $f(x)$ below. Sketch a graph of $f'(x)$:

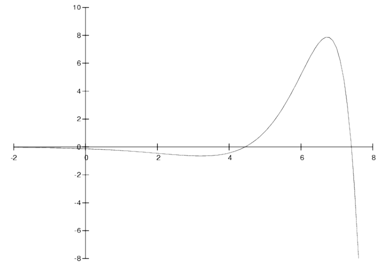


2. Match the following graphs on the left to their derivatives on the right.

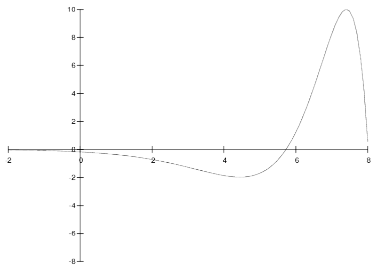
(a)



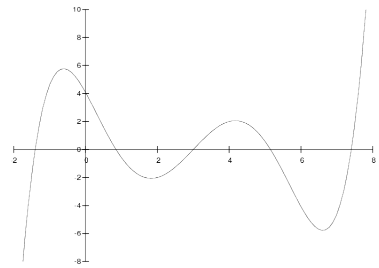
(1)



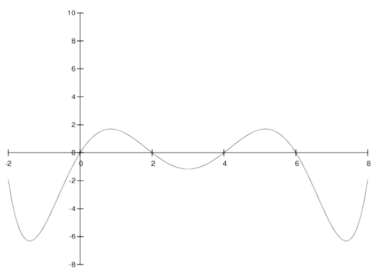
(b)



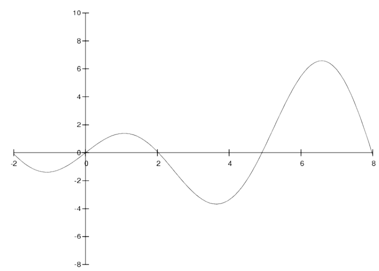
(2)



(c)



(3)



3. Use the graphs from the previous question to fill in the blanks.

    (a) If $f'(x) > 0$ on a interval, then $f(x)$ is ____________________ on that interval.

    (b) If $f'(x) < 0$ on a interval, then $f(x)$ is ____________________ on that interval.

Interpreting the Derivative - Units

Questions

4. Suppose the function $s(t)$ gives the distance due East from Durham of a car traveling along the road.

    (a) What are sensible units for t and for s?

    (b) What are the units of the derivative function of $s(t)$, namely $s'(t)$?

(c) Describe in words what knowing a formula for $s'(t)$ would tell you.

5. Suppose that $P(t)$ gives the population of the United States in millions of people, where t is 'years since 1800'. What are the units of $P'(t)$? What does the expression $P'(213) = 2.1$ mean?

6. Suppose that $q(m)$ thousands of units of a particular product are sold when the price is m dollars.

   (a) Explain the following statements in words, giving units for all your numbers:

      i. $q(10) = 100$.

      ii. $q'(10) = -3$.

   (b) Write the following in mathematical notation:

      i. If the price of the product is low (less than $5 per item), increasing it increases demand, as people think it is of higher quality.

      ii. However, if the price is too high (greater than $5 per item), increasing it further decreases demand, as people think it is too expensive.

   (c) If the data in part (b) of this question is true, what is $q'(5)$? Explain.

   (d) If you were given a graph of $q'(m)$, explain how you would find the price that maximizes the number of units sold. Justify your answer.