From Last Time...

- Average velocity on $[a, a+h]$ for an object whose position is given by $s(t)$:

- Instantaneous velocity at a for an object whose position is given by $s(t)$:

- What do they each mean graphically?

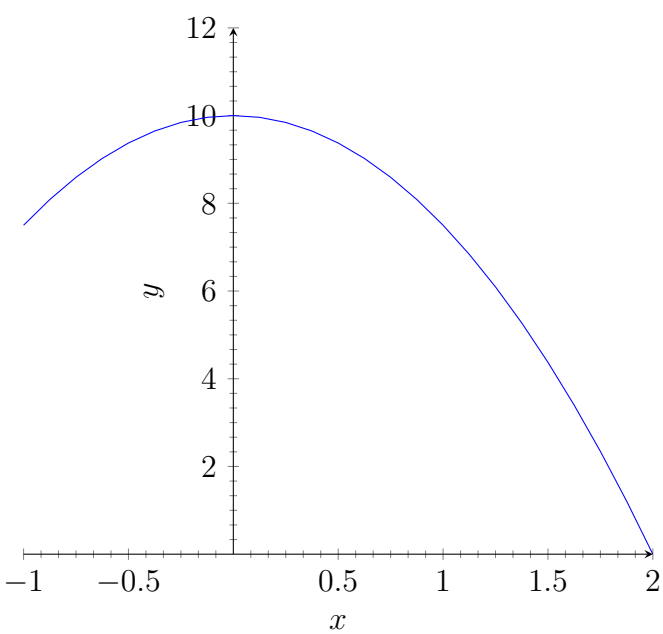
Preliminaries: Average Rate of Change of a Function

The average rate of change of a function $f(x)$ over the interval $[a, a+h]$ is

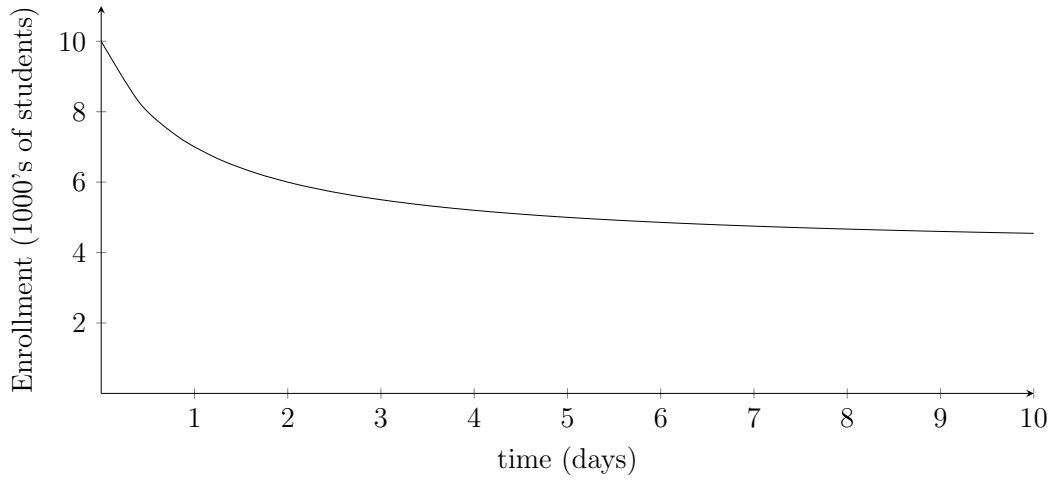
$$\frac{\Delta f}{\Delta x} = \frac{f(a+h) - f(a)}{h}$$

This ratio is also called a **difference quotient**.

On the graph below, illustrate the average rate of change of the function over the interval $[-0.5, 1.5]$.



At MadeUp University, which is absolutely terrible, the enrollment in thousands of students at a time t days after the start of term is given by $E(t) = \frac{6}{t+1} + 4$:



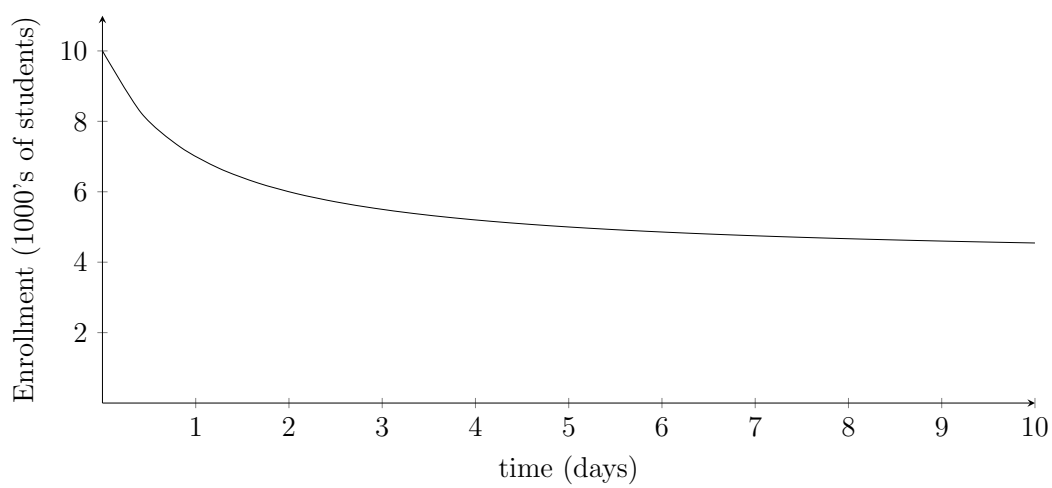
How many thousands of students left during the first week? On average, how many left per day during that time? Include units! Illustrate the average on the graph above and label it.

Instantaneous Rate of Change of a Function

The instantaneous rate of change of a function $f(x)$ at a point $x = a$ is the slope of the tangent line to the function at that point. Note that this corresponds to instantaneous velocity if the function gives position.

1. (a) On the following copy of the graph of enrollment at MadeUp University below, illustrate the instantaneous rates of change of enrollment for the zeroth day and the seventh day. Label your answers.

□Regrade



(b) Rank these three quantities in order: i. average rate of change between day zero and day seven; ii. instantaneous rate of of change at time zero; iii. instantaneous rate of change at time seven.

_____ < _____ < _____

(c) By calculating $\frac{E(7+h)-E(7)}{h}$ for small values of h, just as we did before, fill in the table to estimate the instantaneous rate of change of enrollment at 7 days to three decimal places. Show *one* of your computations in full.

h (days)	Avg. RoC on $[7, 7+h]$	h (days)	Avg. Vel on $[7, 7+h]$
-0.1		0.1	
-0.01		0.01	
-0.001		0.001	

The Derivative

Given a function $f(x)$, we define its **derivative at the point** $x = a$ to be

$$f'(a) = \lim_{\rightarrow} \underline{\hspace{4cm}},$$

if this limit exists.

This can be interpreted as the instantaneous rate of change of f at the point $x = a$. It also the slope of the curve $y = f(x)$ at the point $(a, f(a))$, or alternatively, the slope of the tangent line to the curve at this point.

2. Consider the function given by $y = -3x^2 + 4$.

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(a) Estimate the slope of the tangent line to the curve $y = -3x^2 + 4$ at $x = 1$ by using the table method to compute an estimate for $f'(1)$. Again, show *one* of your computations.

h	Avg. RoC on $[1, 1+h]$	h	Avg. Vel on $[1, 1+h]$
-0.1		0.1	
-0.01		0.01	
-0.001		0.001	

(b) Find $f'(1)$ exactly by computing the limit algebraically.

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} =$$

(c) Next, we're going to find the equation of the tangent line to the curve when $x = 1$:

i. What is the slope of the tangent line to the curve $y = -3x^2 + 4$ at $x = 1$? (Hint: you already did the work.)

ii. What point must be on the tangent line? (Hint: the tangent has to touch the curve when $x = 1$. What is the y-coordinate?)

iii. Find the equation of the tangent line to the graph at the point 1. Be sure to graph both the function and your answer on your calculator to check that you're right.

3. A graph of the same function is shown in the three questions below. Which of the given pair is larger in each of the following pairs? Illustrate each pair on the graph provided.

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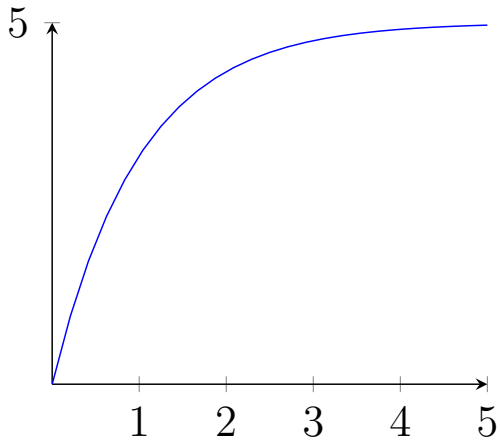
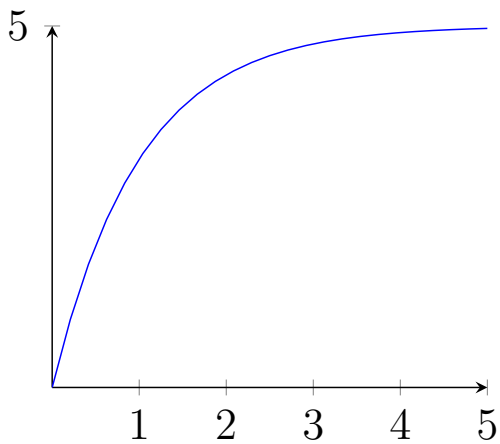
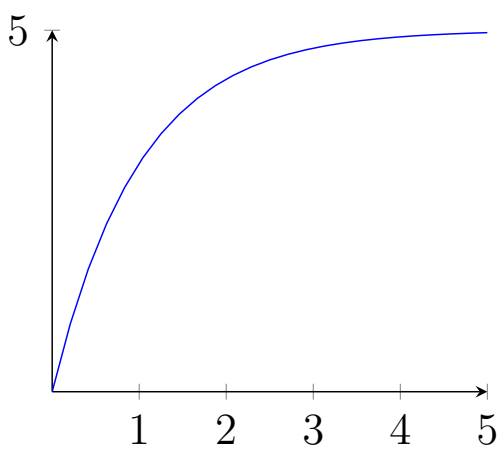
(a) The average rate of change of the function on $[1, 3]$ or the average rate of change of the function on $[3, 5]$?



(b) $f(2)$ or $f(4)$?



(c) $f'(2)$ or $f'(4)$?