From Last Time...

I Average velocity on $[a, a+h]$ for an object whose position is given by $s(t)$:

II Instantaneous velocity at a for an object whose position is given by $s(t)$:

III What do they each mean graphically?

Average Rate of Change of a Function

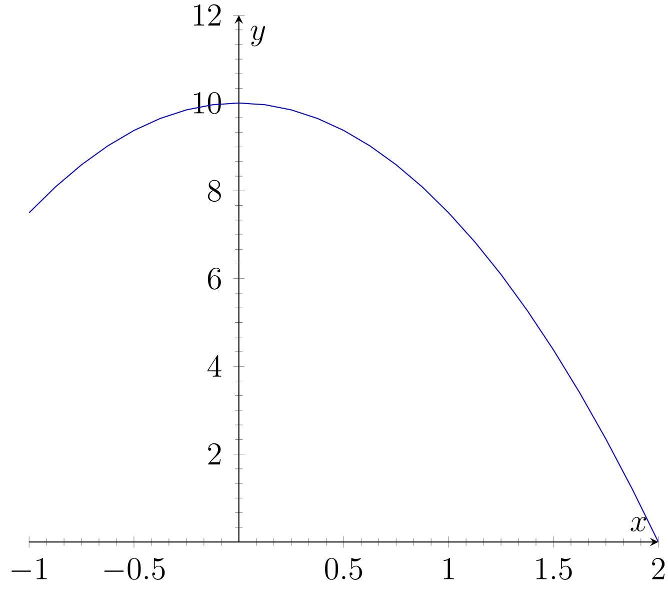
The average rate of change of a function $f(x)$ over the interval $[a, a+h]$ is

$$\frac{\Delta f}{\Delta x} = \underline{\hspace{3cm}}$$

This ratio is also called a __________________ __________________.

Questions

1. On the graph below, illustrate the average rate of change of the function over the interval $[-0.5, 1.5]$.



2. At MadeUp University, the enrollment in thousands of students at a time t days after the start of term is given by
$$E(t) = \frac{3}{t+1} + 7.$$
What is $E(0)$? Where does $E(t)$ have a horizontal asymptote? Using this information, sketch $E(t)$ for $t > 0$, labelling both these pieces of information.

   (a) How many thousands of students left during the first week? On average, how many left per day during that time? Include units! Illustrate the average on the graph above.

   (b) Illustrate the instantaneous rates of change of enrollment for the zeroth day and the seventh day on the graph above. Label your answers.

   (c) Rank these three quantities in order: i. average rate of change between day zero and day seven; ii. instantaneous rate of of change at time zero; iii. instantaneous rate of change at time seven.

   ____ ____ ____

   (d) Use the table to estimate the instantaneous rate of change of enrollment at 7 days to three decimal places. (Hint: calculate $\frac{E(7+h)-E(7)}{h}$ for some small values of h, just as we did before.)

Instantaneous Rate of Change, a.k.a Derivative

Given a function $f(x)$, we define its **derivative at the point** $x = a$ to be

$$f'(a) = \lim_{\_\_ \to \_\_} \_\_\_\_\_\_\_\_,$$

if this limit exists.

This can be interpreted as the instantaneous rate of change of f at the point $x = a$. It also the slope of the curve $y = f(x)$ at the point $(a, f(a))$, or alternatively, the slope of the tangent line to the curve at this point.

Questions

3. Consider the function given by $y = -3x^2 + 4$.

   (a) Estimate the slope of the tangent line to the curve $y = -3x^2 + 4$ at $x = 1$ by using the table method to compute an estimate for $f'(1)$.

   (b) Find $f'(1)$ exactly by computing the limit algebraically.

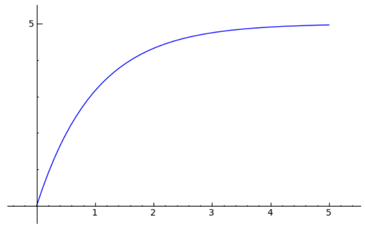
   (c) Next, we're going to find the equation of the tangent line to the curve when $x = 1$:

      i. What is the slope of the tangent line to the curve $y = -3x^2 + 4$ at $x = 1$? (Hint: you already did the work.)

      ii. What point must be on the tangent line? (Hint: the tangent has to touch the curve when $x = 1$. What is the y-coordinate?)

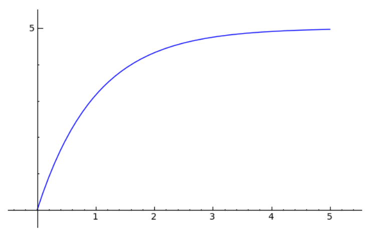
      iii. Find the equation of the tangent line to the graph at the point 1. Be sure to graph both the function and your answer on your calculator to check that you're right.

4. Consider the curve of a function $f(x)$ shown in the picture below.

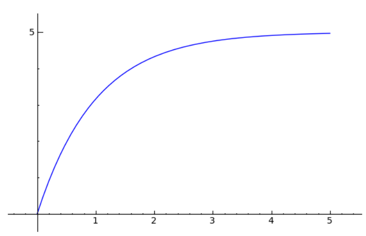


Which is larger in each of the following pairs? Explain and illustrate each pair on the graphs provided in each question:

(a) The average rate of change of the function on $[1, 3]$ or the average rate of change of the function on $[3, 5]$?



(b) $f(2)$ or $f(4)$?



(c) $f'(2)$ or $f'(4)$?