Reminder: Speed vs. Velocity

If an object is moving along a straight line, and we pick one direction to be the positive direction, then the object's *velocity* is positive if it is moving in that direction, and negative if it is moving in the opposite direction. Velocity can be positive or negative, whereas *speed* is always non-negative.

Speed is the _____ of velocity

Preliminaries: Average Velocity

Average Velocity from Data

Suppose we drop a watermelon from the top of Duke Chapel and measure its position every half second, as well as the time it hits the ground. The results are recorded in the following table:

time(sec)	0	.5	1.0	1.5	2.0	2.5	2.80
position(feet)	64.0	62.9	57.3	47.2	32.5	13.4	0.0

- Calculate the average velocity of the watermelon between $t = 0$ and $t = 1$. What is its average speed?

- Calculate the average velocity of the watermelon between $t = 1$ and $t = 1.5$ seconds.

Average Velocity from a Position Function

Suppose an object is traveling along a straight line. Its distance (in meters) from its starting point at time t seconds is given by $s(t) = -4t(t-1)$. What is the average velocity of the object between:

- $t = 0$ and $t = 1$?

- $t = 0$ and $t = 0.5$?

- $t = 0$ and $t = 0.1$?

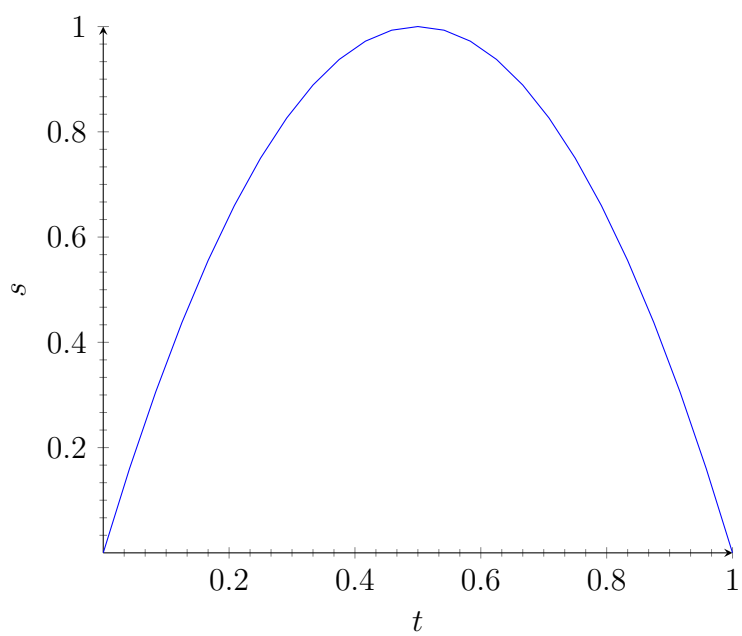
Average Velocity

If the position of an object at time t is given by a function $s(t)$, then the average velocity of the object over a time interval $[a, b]$ is given by

$$\text{Average Velocity} = \underline{\hspace{3cm}} = \underline{\hspace{3cm}}$$

The formula for average velocity looks like the formula for a slope!

On the following graph of $s(t) = -4t(t-1)$, draw and label lines whose slopes correspond to the three average velocities you computed above:



Instantaneous Velocity

All of the calculations above computed the *average* velocity *between two times points*. What if we wanted to know how fast the particle was going *at a particular point in time*? This is called the particle's ***instantaneous velocity*** at that point in time.

1. Why can't we use the formula $\frac{s(b)-s(a)}{b-a}$ to compute instantaneous velocity? What □Regrade happens to that formula if $a = b$?

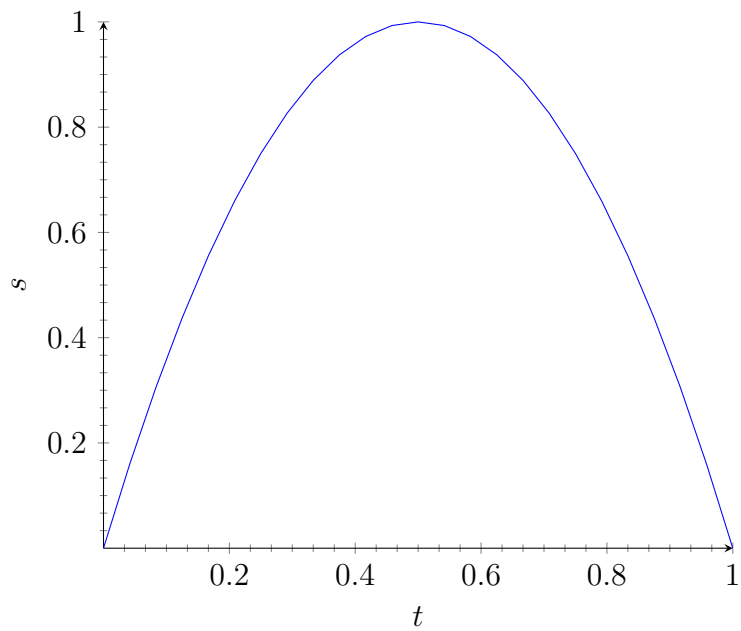
2. We need a different approach to compute the instantaneous velocity of the object in the previous question at, say, $t = 0$ seconds. Let's compute the *average* velocity of the particle between $t = 0$ and $t = h$ seconds for smaller and smaller values of h:

□Regrade

(a) Do this for the values of h in the table:

h (sec)	Avg. Vel on $[0, h]$ (m/s)
0.1	
0.01	
0.001	

(b) What appears to be the instantaneous velocity of the object at $t = 0$ seconds? On the graph of $s(t)$ below, draw a line touching $(0, s(0)) = (0, 0)$ whose slope is this velocity and label it.



If the position of an object at time t is given by a function $s(t)$, then the *instantaneous velocity* at time $t = a$ is given by

Instantaneous Velocity at time $a =$ _____

If we draw the graph of $s(t)$, then the instantaneous velocity of the object at time $t = a$ is given by the _____ of the _____ to the curve at the point (_____, _____).

3. Suppose that an object is moving along a straight line with position (in meters) from ☐Regrade a fixed reference point at time t seconds given by $s(t) = t^2 - 6$.

(a) Calculate the average velocity of the object between $t = 2$ and $t = 2 + h$ for the following values of h:

h (sec)	Avg. Vel on $[2, 2+h]$ (m/s)	h (sec)	Avg. Vel on $[2, 2+h]$ (m/s)
-0.1		0.1	
-0.01		0.01	
-0.001		0.001	

(b) Use your answers to the previous question to *estimate* the instantaneous velocity of the object at time $t = 2$ seconds.

(c) *Our very first true calculus computation!*

Use the limit definition of instantaneous velocity above to *compute* the exact velocity of the object at time $t = 2$ seconds.