Reminder: Speed vs. Velocity

If an object is moving along a straight line, and we pick one direction to be the positive direction, then the object’s velocity is positive if it is moving in that direction, and negative if it is moving in the opposite direction. Velocity can be positive or negative, whereas speed is always non-negative.

Speed is the _____ _____ of velocity

Average Velocity

1. Suppose we drop a watermelon from the top of Duke Chapel and measure its position every half second, as well as the time it hits the ground. The results are recorded in the following table:

<table>
<thead>
<tr>
<th>time(sec)</th>
<th>0</th>
<th>.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
<th>2.80</th>
</tr>
</thead>
<tbody>
<tr>
<td>position(feet)</td>
<td>64.0</td>
<td>62.9</td>
<td>57.3</td>
<td>47.2</td>
<td>32.5</td>
<td>13.4</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Questions

(a) Calculate the average velocity of the watermelon between \( t = 0 \) and \( t = 1 \). What is its average speed?

(b) Calculate the average velocity of the watermelon between \( t = 1 \) and \( t = 1.5 \) seconds.

Average Velocity

If the position of an object at time \( t \) is given by a function \( s(t) \), then the average velocity of the object over a time interval \([a, b]\) is given by

\[
\text{Average Velocity} = \frac{\Delta s}{\Delta t} = ________
\]
2. Suppose an object is traveling along a straight line. Its distance (in meters) from its starting point at time \( t \) seconds is given by \( s(t) = -4t(t - 1) \).

(a) What is the average velocity of the object between:

i. \( t = 0 \) and \( t = 1 \)?
ii. \( t = 0 \) and \( t = 0.5 \)?
iii. \( t = 0 \) and \( t = 0.1 \)?

(b) On the following graph of \( s(t) = -4t(t - 1) \), draw and label lines whose slopes correspond to the three average velocities you computed above:

![Graph of \( s(t) = -4t(t - 1) \)]

### Instantaneous Velocity

3. Suppose now that we wanted to compute the instantaneous velocity of the object in the previous question at time \( t = 0 \) seconds.

(a) Do this for the values of \( h \) in the table:

<table>
<thead>
<tr>
<th>( h ) (sec)</th>
<th>Avg. Vel on ([0, h]) (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>0.001</td>
<td></td>
</tr>
</tbody>
</table>

(b) Approximately what is the instantaneous velocity of the object at \( t = 0 \) seconds? Draw a line whose slope is this velocity on the graph above and label it.
If the position of an object at time $t$ is given by a function $s(t)$, then the *instantaneous velocity* at time $t = a$ is given by

\[
\text{Instantaneous Velocity at time } a = \lim_{h \to 0} \frac{s(a + h) - s(a)}{h}
\]

If we draw the graph of $s(t)$, then the instantaneous velocity of the object at time $t = a$ is given by the ________ of the curve at the point (___, ___).

4. Suppose that an object is moving along a straight line with position (in meters) from a fixed reference point at time $t$ seconds given by $s(t) = t^2 - 6$.

(a) Calculate the average velocity of the object between $t = 2$ and $t = 2 + h$ for the following values of $h$:

<table>
<thead>
<tr>
<th>$h$ (sec)</th>
<th>Avg. Vel on $[2, 2 + h]$ (m/s)</th>
<th>$h$ (sec)</th>
<th>Avg. Vel on $[2, 2 + h]$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-0.1$</td>
<td></td>
<td>$0.1$</td>
<td></td>
</tr>
<tr>
<td>$-0.01$</td>
<td></td>
<td>$0.01$</td>
<td></td>
</tr>
<tr>
<td>$-0.001$</td>
<td></td>
<td>$0.001$</td>
<td></td>
</tr>
</tbody>
</table>

(b) Use your answers to the previous question to estimate the instantaneous velocity of the object at time $t = 2$ seconds.

(c) Use the limit definition of instantaneous velocity above to compute the exact velocity of the object at time $t = 2$ seconds.