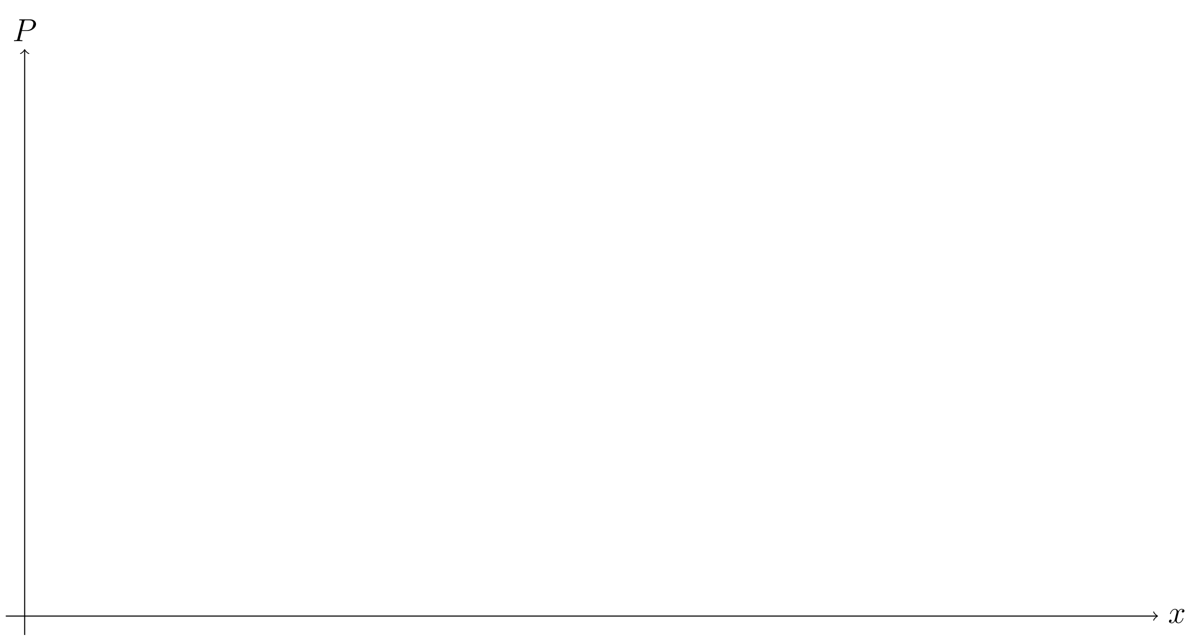
This worksheet consists of questions that explore some mathematical modeling, an essential part of this class. We will use many of the ideas throughout the course. Be sure to complete all questions as homework if you don't finish in class!

1. Suppose that a person meets their future partner at age 18, that the partner dies when the person is 70 and that the person then goes on to live to age 100. If x represents the person's age, draw the graph showing the proportion $P(x)$ of their life that they have been with their partner. Then write down a formula for $P(x)$, showing all your work.

$$P(x) = \begin{cases} \underline{\hspace{3cm}} & \text{if } x < 18 \\ \underline{\hspace{3cm}} & \text{if } 18 \leq x \leq 70 \\ \underline{\hspace{3cm}} & \text{if } 70 < x \leq 100 \end{cases}$$



2. (a) Suppose you have a car that gets 24 miles per gallon and that you are going to buy 10 gallons of gas. A gas station near you sells gas for \$2.499 but one 4 miles down the road sells gas for \$2.459. Assuming that the cost of the gas is the only extra cost, would you save money by driving the extra four miles?

   (b) What if the gas station were only one mile away?

   (c) What is the least distance at which you would not save any money?

   (d) Let

   $$\begin{aligned} E &= \text{the miles per gallon your car gets} \\ P_1 &= \text{the price at the less expensive gas station} \\ P_2 &= \text{the price at the more expensive gas station} \\ M &= \text{the number of additional miles to the less expensive gas station} \\ G &= \text{the number of gallons of gas you would buy (at either station)} \end{aligned}$$

   Write a forumla that gives the savings (positive or negative) of driving to the less expensive gas station if you drive M additional miles.

   (e) Write a forumula for M in terms of E, P_1, P_2 and G when there are no savings. i.e. write a formula for the maximum number of miles before it is not cost effective to drive to the cheaper gas station.

3. Consider a situation in which a contagious disease is spreading through a a city of 1,000,000 people. Assume also that:

   (a) no one (or almost no one) dies from the disease,

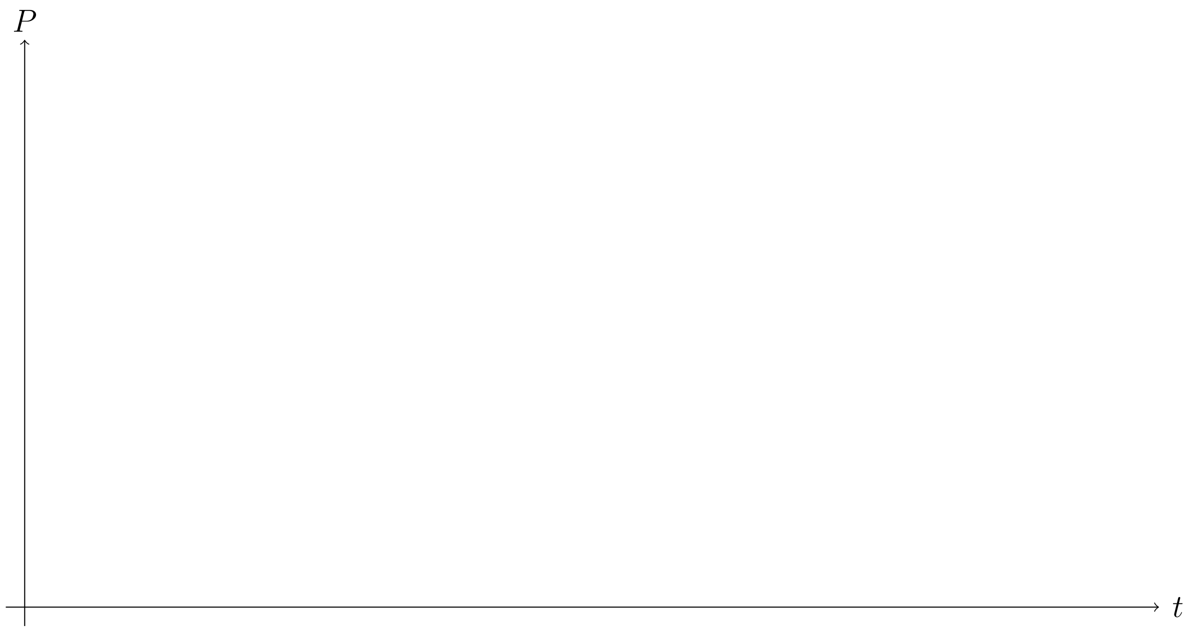
   (b) that once one recovers from it, one doesn't get the disease again,

   (c) that the epidemic runs its course in a one month period,

   (d) that 20% of the population never gets the disease, and

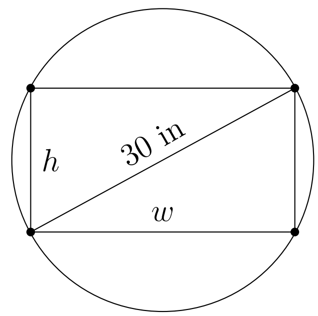
   (e) the the epidemic begins with one person.

   Draw a possible graph that gives the number of people who *have contracted* the disease during the epidemic as a function of time. Label your axes carefully (including the scale used on each axis) and explain in a few complete sentences in a coherent paragraph why your graph is consistent with the assumptions given above. In particular, justify the shape of the graph.



4. (From the Introductory Mathematical Modeling course used and developed at USMA at West Point) You are working with an aerial photograph taken by an unmanned aerial vehicle (UAV) of a meeting between an unknown enemy operative and a known enemy operative. The meeting was outside in bright sunlight. The known operative's height is 5 feet, 8 inches, and the length of his shadow on the ground is three feet. The length of the unknown operative's shadow is 3 feet, 3 inches. You would like to know his height to help identify him at a later time. Find the height of the unknown operative.

5. A rectangular beam is cut from a cylindrical log of diameter 30 inches (see picture). The strength of a beam of width w and height h is proportional to wh^2. If the strength is 146.25 pounds when $h = 18$ inches and $w = 24$ inches, find a formula for the strength in terms of w. what is the proportionality constant and what are its units?



6. Police sometimes use the formula $s = \sqrt{24d}$ to estimate the speed s in miles per hour of a car driving on dry concrete pavement that leaves a skid mark of length d feet. Complete the following sentences: s is proportional to ______. If you are driving at 60 miles per hour and slam on the breaks, how long will your skid marks be?

7. A wire 6 meters long is cut into twelve pieces, eight of length x and four of length y. These pieces are welded together to form a box with a square base. What are the dimensions of the box in terms of x and y? What is the volume of the box in terms of x?

8. In a certain orchard, the following facts are known:
   - if 24 trees are planted per acre, each tree will yield 600 apples and
   - for each additional tree planted per acre, the yield decreases by 12 apples per tree.

   (a) Let x be the number of trees per acre. In terms of x, what is A, the number of apples per tree? (See Worksheet 1-3!)

   (b) In terms of x, what is Y, the yield of apples/acre? (Hint: You already have the number of apples per tree...what should you do to that to get the number of apples/acre?)

   (c) Draw a graph of Y vs x. At what number of trees/acres does the yield drop to zero? What is the optimal number of trees per acre that will maximize total apple yield?

9. A pebble is dropped in still water, forming a circular ripple whose radius is expanding at a constant rate of 10 cm/sec. Find a formula giving the area enclosed by the ripple as a function of time.