Preliminaries

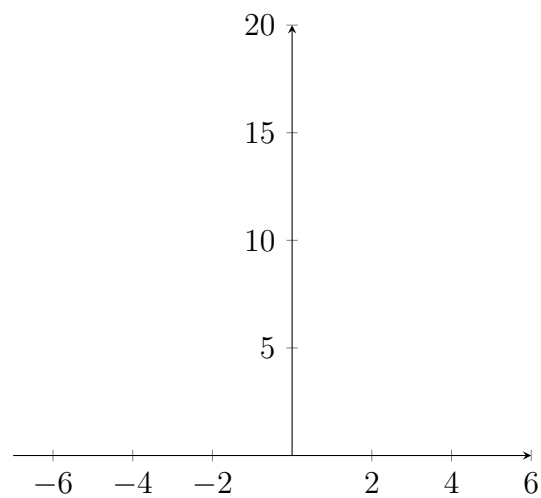
There are three main ways in which we can transform the graph of a function:

- Shifting - up/down or left/right.
- Stretching/compressing - in the horizontal or vertical directions.
- Reflecting - in the x- or y-axis.

The first two correspond to addition and multiplication respectively. The last is also multiplication, but by a negative number.

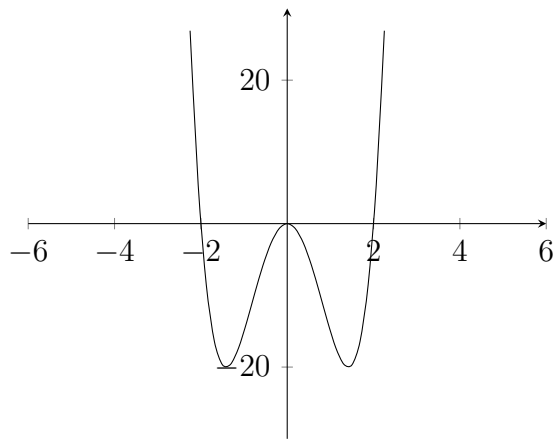
Transforming Functions by Addition: Shifts

- Suppose $f(x) = x^2$. Then write out expressions for:
  - $f(x) + 2$
  - $f(x + 2)$
- On the axes below, draw the graphs of $f(x)$, $f(x)+2$ and $f(x+2)$. Label your graphs.



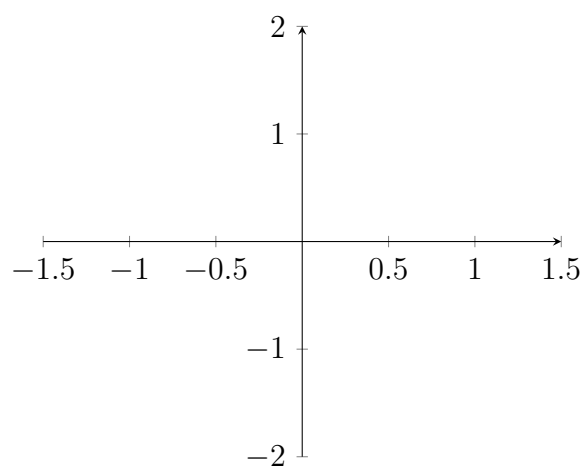
- Fill in the blanks:
  - If we start from the graph of a function $f(x)$, the graph of the function $f(x) + a$ has exactly the same shape, but is shifted _____ by _____ units. If a is negative, the functions shifts _____.
  - If we start from the graph of a function $f(x)$ the graph of the function $f(x + a)$ has exactly the same shape, but is shifted _____ by _____ units. If a is negative, the functions shifts _____.

- Given the graph of a function $f(x)$ below, draw the functions $f(x+3)$ and $f(x)-8$ on the same axes.



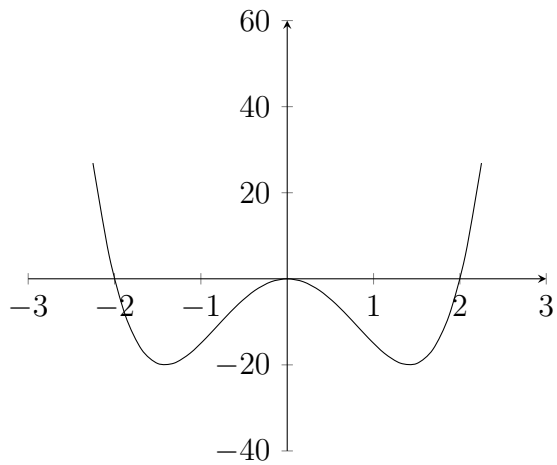
Transforming Functions by Multiplication: Stretches/Compressions

- Suppose $f(x) = x(x-1)(x+1)$. Then write out expressions for:
  - $f(2x)$
  - $2f(x)$
- On the axes below, draw the graphs of $f(x)$, $f(2x)$ and $2f(x)$. Label your graphs.



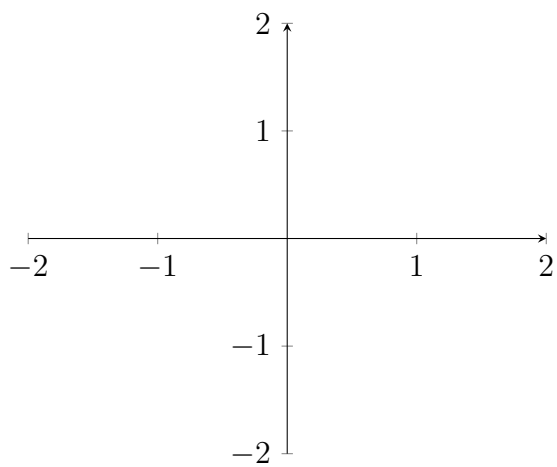
- Fill in the blanks:
  - If we start from the graph of a function $f(x)$, the graph of the function $af(x)$ has the same shape, but is stretched _____ by a factor of _____. If a is a fraction, the graph is _____.
  - If we start from the graph of a function $f(x)$ the graph of the function $f(ax)$ has exactly the same shape, but is compressed _____ by a factor of _____. If a is a fraction, the graph is _____.

- Given the graph of a function $f(x)$ below, draw the functions $f(2x)$ and $2f(x)$ on the same axes.



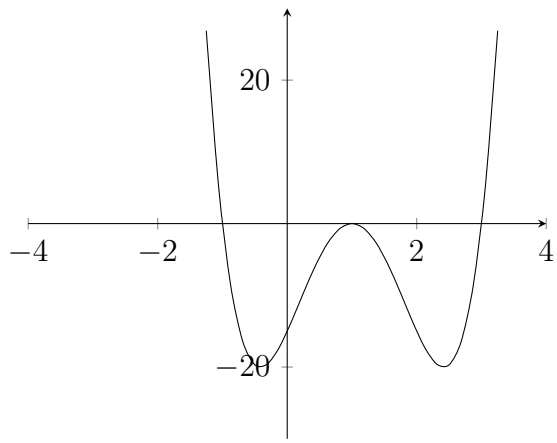
Transforming Functions by Reflection

- Suppose $f(x) = x(x-1)$. Then write out expressions for:
  - $f(-x)$
  - $-f(x)$
- On the axes below, draw the graphs of $f(x)$, $-f(x)$ and $f(-x)$. Label your axes and graphs.



- Fill in the blanks:
  - If we start from the graph of a function $f(x)$, the graph of the function $-f(x)$ is the same graph, but _____ in the _____-axis.
  - If we start from the graph of a function $f(x)$, the graph of the function $f(-x)$ is the same graph, but _____ in the _____-axis.

- Given the graph of the function $f(x)$ below, draw the functions $-f(x)$ and $f(-x)$ on the same axes.



Putting it all Together

1. Consider the graph of $f(x)$ in the previous question.

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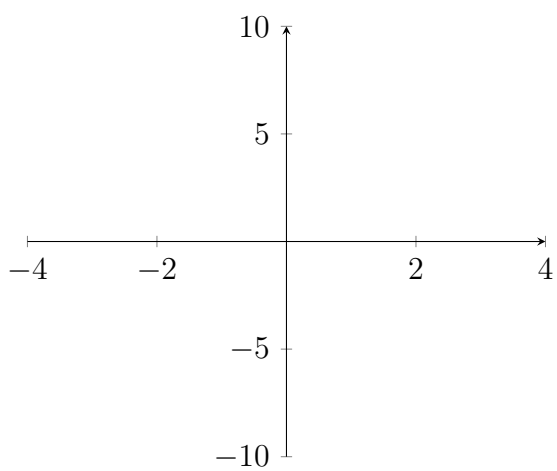
(a) What are its roots? Can you figure out a possible formula for it? (Hint: think back to polynomials.)

(b) Write down a formula for $f(2(x-1))$. Simplify it, but do not FOIL. What are the roots of $f(2(x-1))$?

(c) On the axes below, draw the graphs of $f(x)$ and $f(2(x-1))$.



(d) By considering the zeros of $f(x)$ in part (a), of $f(2(x-1))$ in part (b), and the graphs in part (c), decide which of the following statements is true and which is false:

i. To get from $f(x)$ to $f(2(x-1))$ you first compress along the x-axis by a factor of 2, then shift to the right by 1.

ii. To get from $f(x)$ to $f(2(x-1))$ you first shift to the right by 1, then compress along the x-axis by a factor of 2

2. Given a graph of $f(x)$, describe how you would go about drawing the graph of $f\left(\frac{1}{2}x-3\right)$. □Regrade

Transforming By a Linear Function

To transform the graph of $f(x)$ to the graph of $f(ax+b)$:

- Factor $ax+b$: $f(ax+b) = f(a(x+c))$.
- Compress by a factor of a (or stretch by a factor of $\frac{1}{a}$ if a is a fraction).
- Shift left by c (or right if c is negative).

The order matters!

3. Given the graph of $f(x)$ below, draw the graph of $-2f(0.5x)-3$. Do this step-by-step using the axes below. Be sure to label each with what you're drawing! If you do this correctly, you should end up with a familiar function. Can you use that to identify the function of the original graph?

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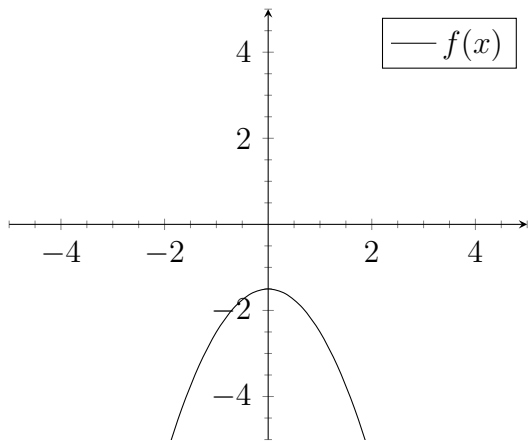

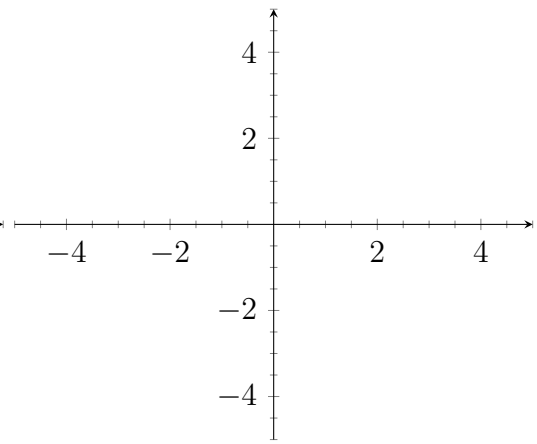

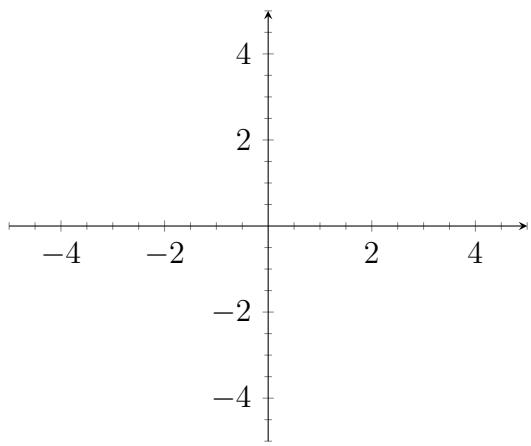

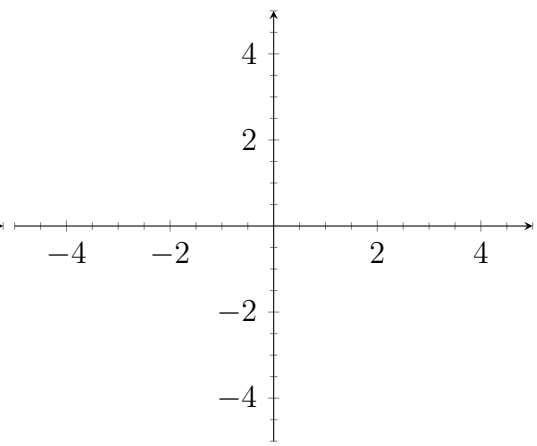

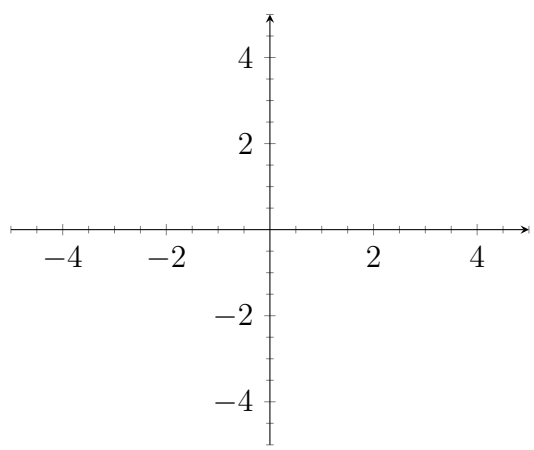

More Problems

The graph of a function $f(x)$ is given in each of the problems below. In each problem, draw (on the same set of axes) the graph of the function(s) obtained from the given transformation(s) of $f(x)$. In problems 7-9, multiple transformations are required. I suggest doing them step by step on scrap paper, then copying the final answer to this worksheet.

4. Graph $f(x+2)$ and $f(x-1)$.

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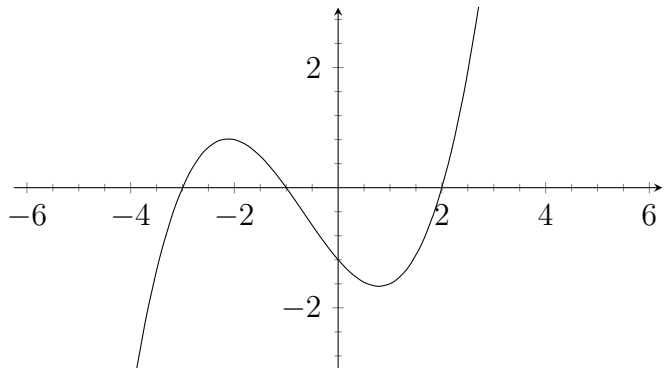

5. Graph $3f(x)$, $0.5f(x)$, and $-2f(x)$.

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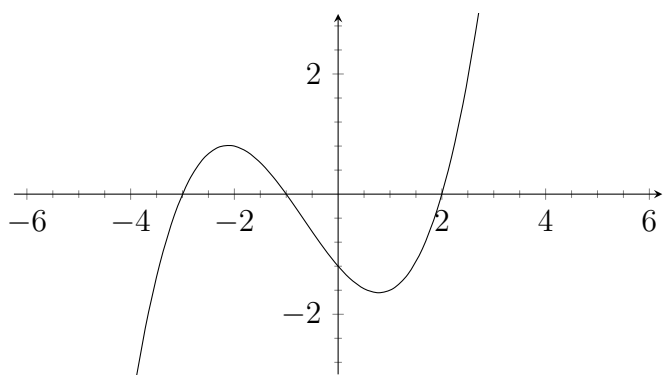

6. Graph $f(3x)$, $f(0.5x)$, and $f(-2x)$.

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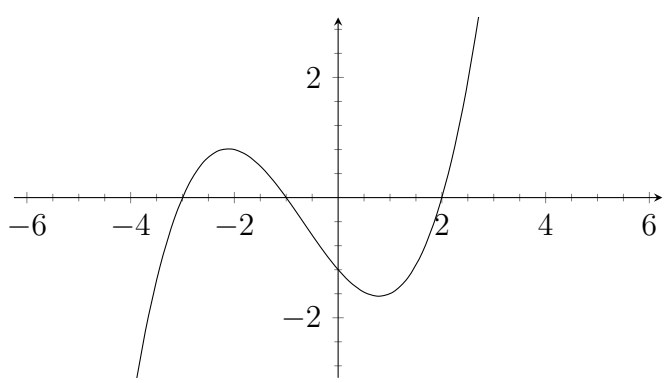

7. Graph $f(0.5x) - 2$.

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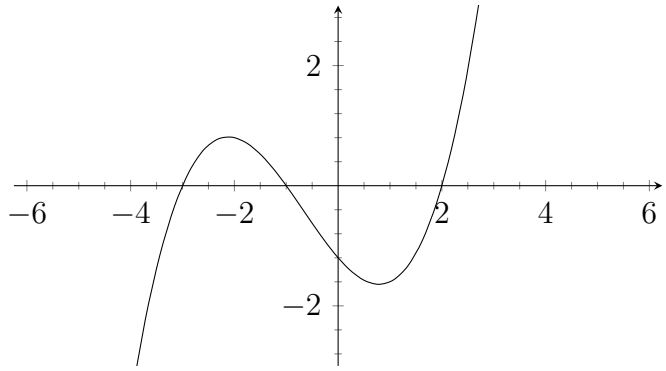

8. Graph $f(2x - 2)$ and $f(2(x - 2))$.

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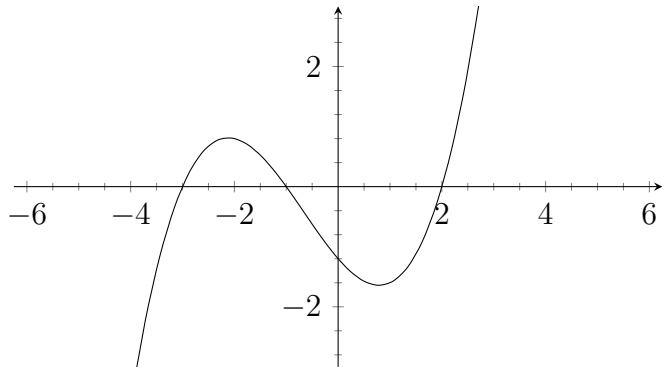
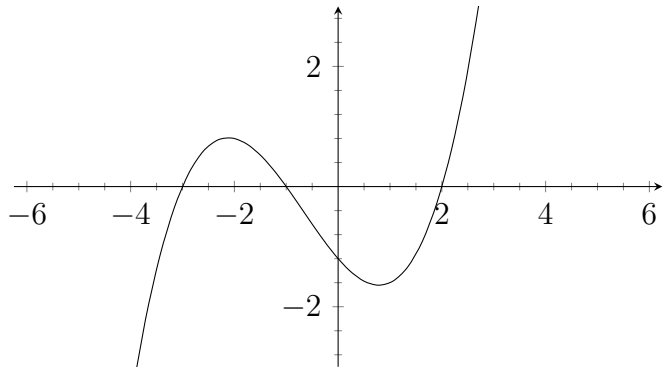


9. Graph $-2f(x + 2) + 1$.

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