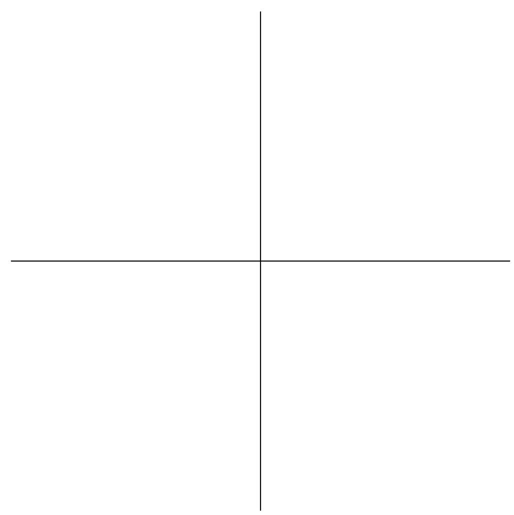
Transforming Functions by Addition

1. Suppose $f(x) = x^2$. Then write out expressions for:

   (a) $f(x) + 2$

   (b) $f(x + 2)$

2. On the axes below, draw the graphs of $f(x)$, $y = f(x) + 2$ and $y = f(x + 2)$. Label your graphs and axes.

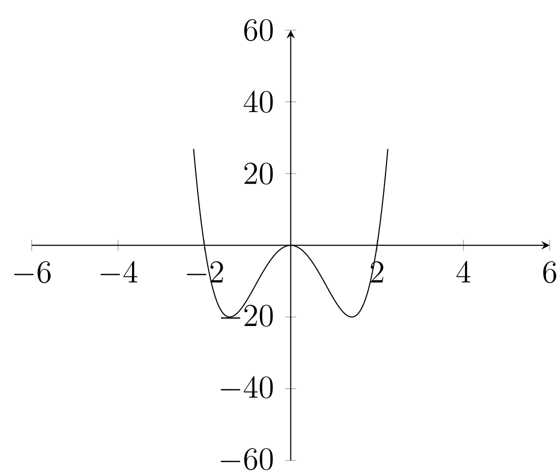


3. Fill in the blanks:

   (a) If we start from the graph of a function $f(x)$, the graph of the function $f(x) + a$ has exactly the same shape, but is shifted _____________ by ______ units.

   (b) If we start from the graph of a function $f(x)$ the graph of the function $f(x + a)$ has exactly the same shape, but is shifted _____________ by ______ units.

4. Given the graph of a function $f(x)$ below, draw the functions $f(x + 3)$ and $f(x) - 8$ on the same axes.



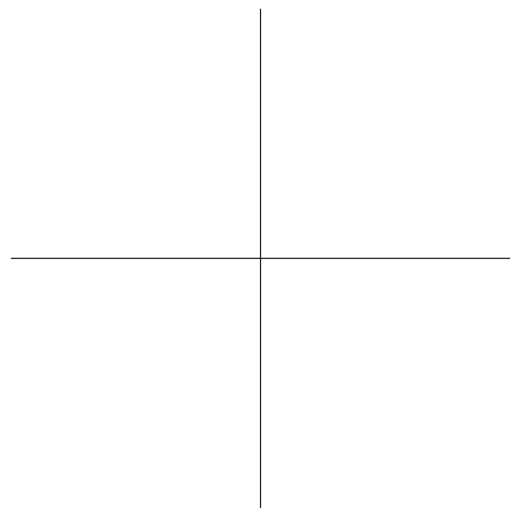
Transforming Functions by Multiplication

5. Suppose $f(x) = x(x-1)(x+1)$. Then write out expressions for:

   (a) $f(2x)$

   (b) $2f(x)$

6. On the axes below, draw the graphs of $f(x)$, $f(2x)$ and $2f(x)$. Label your axes and graphs.

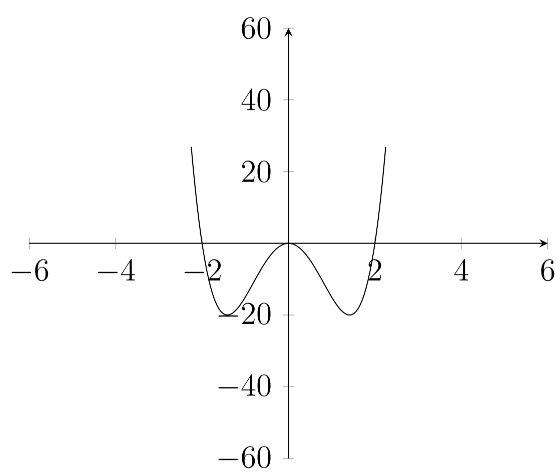


7. Fill in the blanks:

   (a) If we start from the graph of a function $f(x)$, the graph of the function $af(x)$ has the same shape, but is stretched ____________ by a factor of ______.

   (b) If we start from the graph of a function $f(x)$ the graph of the function $f(ax)$ has exactly the same shape, but is stretched ____________ by a factor of ______.

8. Given the graph of a function $f(x)$ below, draw the functions $f(2x)$ and $2f(x)$ on the same axes.



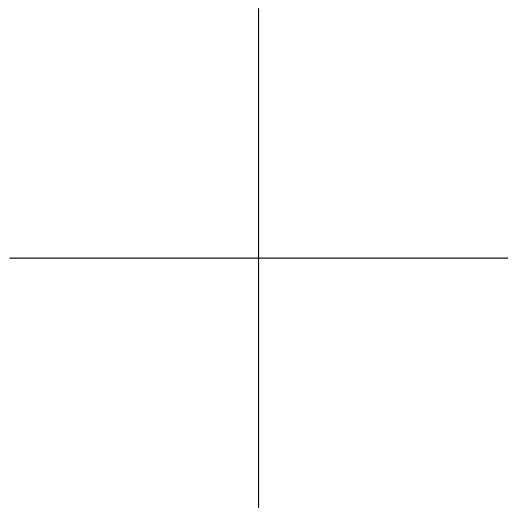
Function Reflections

9. Suppose $f(x) = x(x-1)$. Then write out expressions for:

   (a) $f(-x)$

   (b) $-f(x)$

10. On the axes below, draw the graphs of $f(x)$, $-f(x)$ and $f(-x)$. Label your axes and graphs.

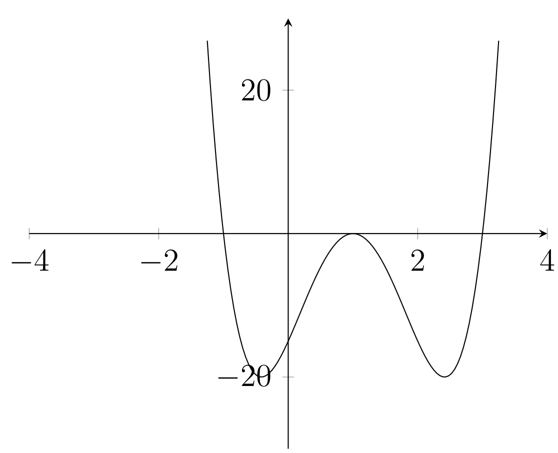


11. Fill in the blanks:

    (a) If we start from the graph of a function $f(x)$, the graph of the function $-f(x)$ is the same graph, but ____________ in the ______-axis.

    (b) If we start from the graph of a function $f(x)$, the graph of the function $f(-x)$ is the same graph, but ____________ in the ______-axis.

12. Given the graph of the function $f(x)$ below, draw the functions $-f(x)$ and $f(-x)$ on the same axes.



Putting it all Together

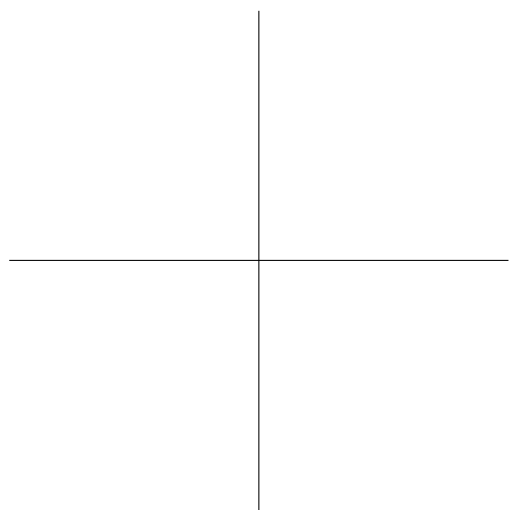
Question

13. Consider the graph of $f(x)$ in the previous question.

    (a) Can you figure out a possible formula for it? (Hint: think back to polynomials.)

    (b) Write down a formula for $f(2(x-1))$. Simplify it, but do not FOIL. What are the roots of $f(2(x-1))$?

    (c) On the axes below, draw the graphs of $f(x)$ and $f(2(x-1))$.



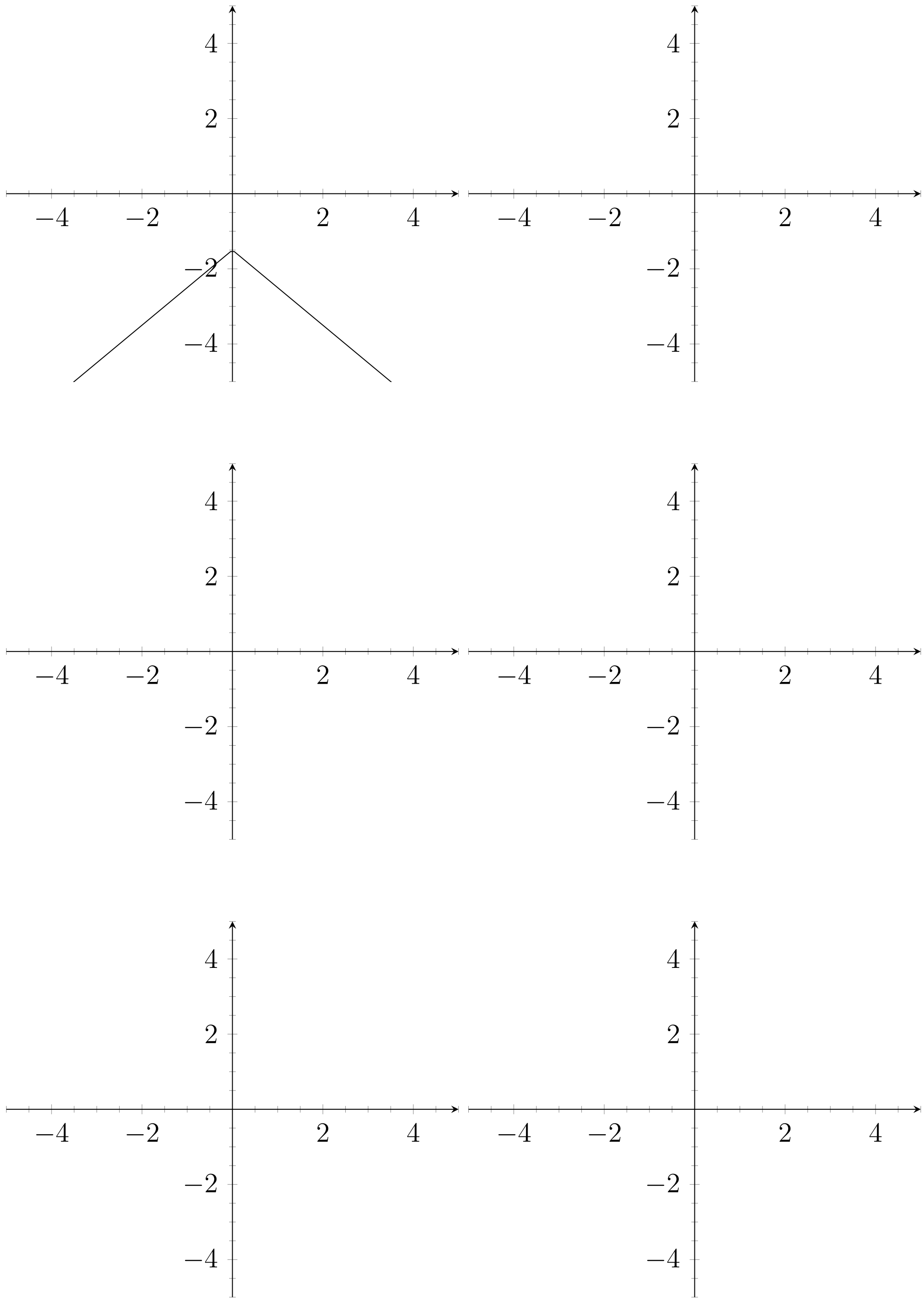
    (d) By considering the zeros of $f(x)$ in part (1), of $f(2(x-1))$ in part (2), and the graphs in part (3), decide which of the following statements is true and which is false:

        i. To get from $f(x)$ to $f(2(x-1))$ you first compress along the x-axis by a factor of 2, then shift to the right by 1.

        ii. To get from $f(x)$ to $f(2(x-1))$ you first shift to the right by 1, then compress along the x-axis by a factor of 2.

14. Given a graph of $f(x)$, describe how you would go about drawing the graph of $f(6x-3)$.

15. Given the graph of $f(x)$ below, draw the graph of $-2f(0.5x) - 3$. (Hint: you might want to do this step-by-step using the axes below. Be sure to label each with what you're drawing!) As a bonus: can you identify (with a formula) the function you get at the end? Can you use that to identify the function of the original graph?

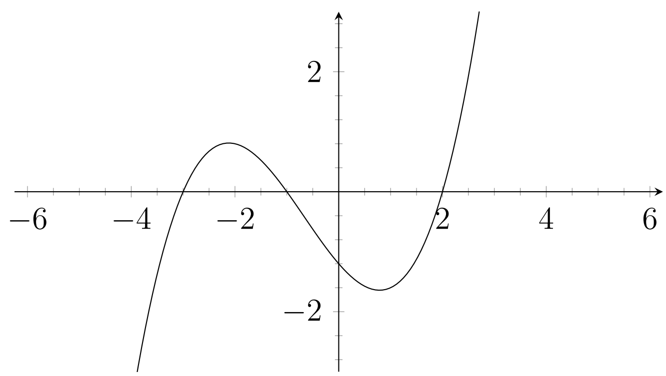


Extra Homework Problems

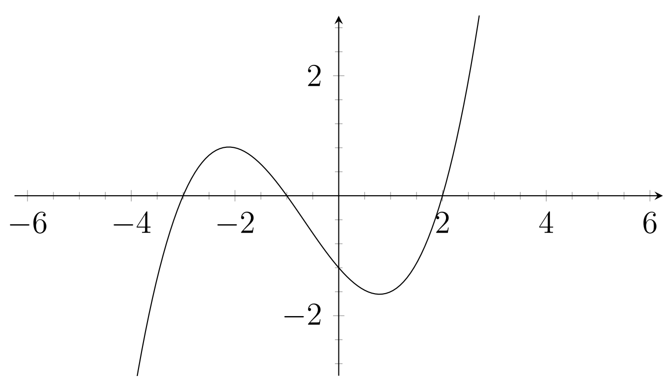
The graph of a function $f(x)$ is given in each of the problems below. In each problem, draw (on the same set of axes) the graph of the function(s) obtained from the given transformation(s) of $f(x)$.

---

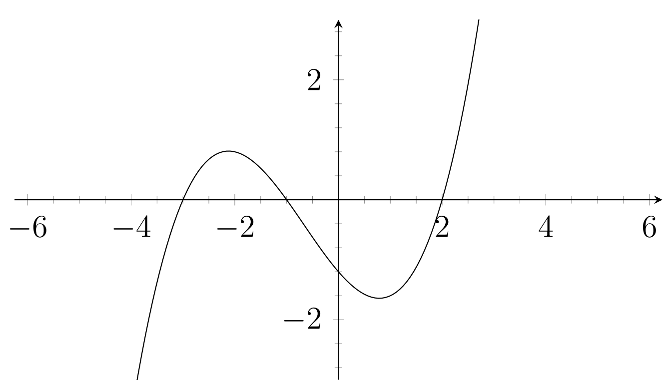
1. Graph $f(x+2)$ and $f(x-1)$.



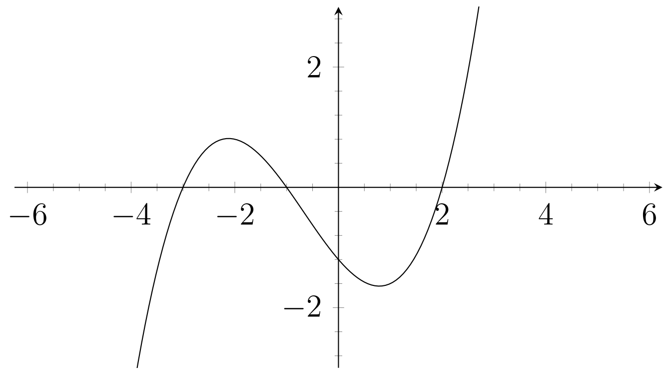
2. Graph $3f(x)$, $0.5f(x)$, and $-2f(x)$.



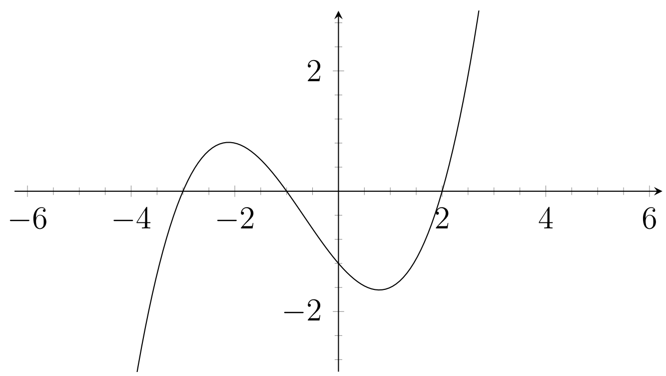
3. Graph $f(3x)$, $f(0.5x)$, and $f(-2x)$.



4. Graph $f(0.5x) - 2$.



5. Graph $f(2x - 2)$ and $f(2(x - 2))$.



6. Graph $-2f(x + 2) + 1$.