Transforming Functions by Addition

1. Suppose \( f(x) = x^2 \). Then write out expressions for:

   (a) \( f(x) + 2 \)

   (b) \( f(x + 2) \)

2. On the axes below, draw the graphs of \( f(x) \), \( y = f(x) + 2 \) and \( y = f(x + 2) \). Label your graphs and axes.

3. Fill in the blanks:

   (a) If we start from the graph of a function \( f(x) \), the graph of the function \( f(x) + a \) has exactly the same shape, but is shifted \______________ \ by \______ \ units.

   (b) If we start from the graph of a function \( f(x) \) the graph of the function \( f(x + a) \) has exactly the same shape, but is shifted \______________ \ by \______ \ units.

4. Given the graph of a function \( f(x) \) below, draw the functions \( f(x + 3) \) and \( f(x) - 8 \) on the same axes.
Transforming Functions by Multiplication

5. Suppose \( f(x) = x(x - 1)(x + 1) \). Then write out expressions for:

(a) \( f(2x) \)
(b) \( 2f(x) \)

6. On the axes below, draw the graphs of \( f(x) \), \( f(2x) \) and \( 2f(x) \). Label your axes and graphs.

7. Fill in the blanks:

(a) If we start from the graph of a function \( f(x) \), the graph of the function \( af(x) \) has the same shape, but is stretched ________ by a factor of ______.

(b) If we start from the graph of a function \( f(x) \) the graph of the function \( f(ax) \) has exactly the same shape, but is stretched ________ by a factor of ______.

8. Given the graph of a function \( f(x) \) below, draw the functions \( f(2x) \) and \( 2f(x) \) on the same axes.
Function Reflections

9. Suppose $f(x) = x(x - 1)$. Then write out expressions for:
   
   (a) $f(-x)$
   
   (b) $-f(x)$

10. On the axes below, draw the graphs of $f(x)$, $-f(x)$ and $f(-x)$. Label your axes and graphs.

11. Fill in the blanks:

   (a) If we start from the graph of a function $f(x)$, the graph of the function $-f(x)$ is the same graph, but _________ in the ____-axis.

   (b) If we start from the graph of a function $f(x)$, the graph of the function $f(-x)$ is the same graph, but _________ in the ____-axis.

12. Given the graph of the function $f(x)$ below, draw the functions $-f(x)$ and $f(-x)$ on the same axes.
Putting it all Together

Question

13. Consider the graph of $f(x)$ in the previous question.

   (a) Can you figure out a possible formula for it? (Hint: think back to polynomials.)

(b) Write down a formula for $f(2(x - 1))$. Simplify it, but do not FOIL. What are the roots of $f(2(x - 1))$?

(c) On the axes below, draw the graphs of $f(x)$ and $f(2(x - 1))$.

(d) By considering the zeros of $f(x)$ in part (1), of $f(2(x - 1))$ in part (2), and the graphs in part (3), decide which of the following statements is true and which is false:

   i. To get from $f(x)$ to $f(2(x - 1))$ you first compress along the $x$-axis by a factor of 2, then shift to the right by 1.

   ii. To get from $f(x)$ to $f(2(x - 1))$ you first shift to the right by 1, then compress along the $x$-axis by a factor of 2.

14. Given a graph of $f(x)$, describe how you would go about drawing the graph of $f(6x - 3)$. 
15. Given the graph of \( f(x) \) below, draw the graph of \(-2f(0.5x) - 3\). (Hint: you might want to do this step-by-step using the axes below. Be sure to label each with what you're drawing!) As a bonus: can you identify (with a formula) the function you get at the end? Can you use that to identify the function of the original graph?
Extra Homework Problems

The graph of a function $f(x)$ is given in each of the problems below. In each problem, draw (on the same set of axes) the graph of the function(s) obtained from the given transformation(s) of $f(x)$.

1. Graph $f(x + 2)$ and $f(x - 1)$.

2. Graph $3f(x)$, $0.5f(x)$, and $-2f(x)$.

3. Graph $f(3x)$, $f(0.5x)$, and $f(-2x)$. 
4. Graph \( f(0.5x) - 2 \).

5. Graph \( f(2x - 2) \) and \( f(2(x - 2)) \).

6. Graph \( -2f(x + 2) + 1 \).