Transforming Functions by Addition

1. Suppose \( f(x) = x^2 \). Then write out expressions for:
   
   (a) \( f(x) + 2 \)
   
   (b) \( f(x + 2) \)

2. On the axes below, draw the graphs of \( f(x) \), \( y = f(x) + 2 \) and \( y = f(x + 2) \). Label your graphs and axes.

3. Fill in the blanks:
   
   (a) If we start from the graph of a function \( f(x) \), the graph of the function \( f(x) + a \) has exactly the same shape, but is shifted \( \underline{\text{__________}} \) by \( \underline{\text{______}} \) units.
   
   (b) If we start from the graph of a function \( f(x) \) the graph of the function \( f(x + a) \) has exactly the same shape, but is shifted \( \underline{\text{__________}} \) by \( \underline{\text{______}} \) units.

4. Given the graph of a function \( f(x) \) below, draw the functions \( f(x + 3) \) and \( f(x) - 8 \) on the same axes.
Transforming Functions by Multiplication

5. Suppose \( f(x) = x(x - 1)(x + 1) \). Then write out expressions for:

   (a) \( f(2x) \)
   (b) \( 2f(x) \)

6. On the axes below, draw the graphs of \( f(x) \), \( f(2x) \) and \( 2f(x) \). Label your axes and graphs.

7. Fill in the blanks:

   (a) If we start from the graph of a function \( f(x) \), the graph of the function \( af(x) \) has the same shape, but is stretched _________ by a factor of _______.

   (b) If we start from the graph of a function \( f(x) \) the graph of the function \( f(ax) \) has exactly the same shape, but is stretched _________ by a factor of _______.

8. Given the graph of a function \( f(x) \) below, draw the functions \( f(2x) \) and \( 2f(x) \) on the same axes.
Function Reflections

9. Suppose \( f(x) = x(x - 1) \). Then write out expressions for:
   
   (a) \( f(-x) \)
   
   (b) \( -f(x) \)

10. On the axes below, draw the graphs of \( f(x) \), \( -f(x) \) and \( f(-x) \). Label your axes and graphs.

11. Fill in the blanks:

   (a) If we start from the graph of a function \( f(x) \), the graph of the function \( -f(x) \) is the same graph, but \( \underline{\phantom{aaa}} \) in the \( \underline{\phantom{aaa}} \)-axis.

   (b) If we start from the graph of a function \( f(x) \), the graph of the function \( f(-x) \) is the same graph, but \( \underline{\phantom{aaa}} \) in the \( \underline{\phantom{aaa}} \)-axis.

12. Given the graph of the function \( f(x) \) below, draw the functions \( -f(x) \) and \( f(-x) \) on the same axes.
Putting it all Together

Question

13. Consider the graph of \( f(x) \) in the previous question.

(a) Can you figure out a possible formula for it? (Hint: think back to polynomials.)

(b) Write down a formula for \( f(2(x - 1)) \). Simplify it, but do not FOIL. What are the roots of \( f(2(x - 1)) \)?

(c) On the axes below, draw the graphs of \( f(x) \) and \( f(2(x - 1)) \).

(d) By considering the zeros of \( f(x) \) in part (1), of \( f(2(x - 1)) \) in part (2), and the graphs in part (3), decide which of the following statements is true and which is false:

   i. To get from \( f(x) \) to \( f(2(x - 1)) \) you first compress along the \( x \)-axis by a factor of 2, then shift to the right by 1.

   ii. To get from \( f(x) \) to \( f(2(x - 1)) \) you first shift to the right by 1, then compress along the \( x \)-axis by a factor of 2.

14. Given a graph of \( f(x) \), describe how you would go about drawing the graph of \( f(6x - 3) \).
15. Given the graph of $f(x)$ below, draw the graph of $-2f(0.5x) - 3$. (Hint: you might want to do this step-by-step using the axes below. Be sure to label each with what you’re drawing!) As a bonus: can you identify (with a formula) the function you get at the end? Can you use that to identify the function of the original graph?
Extra Homework Problems

The graph of a function \( f(x) \) is given in each of the problems below. In each problem, draw (on the same set of axes) the graph of the function(s) obtained from the given transformation(s) of \( f(x) \).

1. Graph \( f(x + 2) \) and \( f(x - 1) \).

2. Graph \( 3f(x) \), \( 0.5f(x) \), and \( -2f(x) \).

3. Graph \( f(3x) \), \( f(0.5x) \), and \( f(-2x) \).
4. Graph $f(0.5x) - 2$.

5. Graph $f(2x - 2)$ and $f(2(x - 2))$.

6. Graph $-2f(x + 2) + 1$. 