Rational Functions

A *rational function* is a function of the form

$$f(x) = \frac{p(x)}{q(x)},$$

where $p(x)$ and $q(x)$ are polynomials.

Examples The following are rational functions:

$$\frac{1}{x}, \frac{x^2-1}{x^4+x^3}, \frac{x^7-8}{x^2+2x-7}.$$

Questions

1. Suppose $f(x) = \frac{p(x)}{q(x)}$ is a rational function. Then

   (a) Suppose $p(a) = 0$ for some number a. Then $f(a) =$ ___, so the graph of $f(x)$ intercepts the ___-axes at $x = a$.

   (b) Suppose $q(b) = 0$ for some number b. Then the graph of $f(x)$ has a _____ _____ at $x = b$.

2. Suppose $r(x) = \frac{x^3-1}{x^2+2x+1}$.

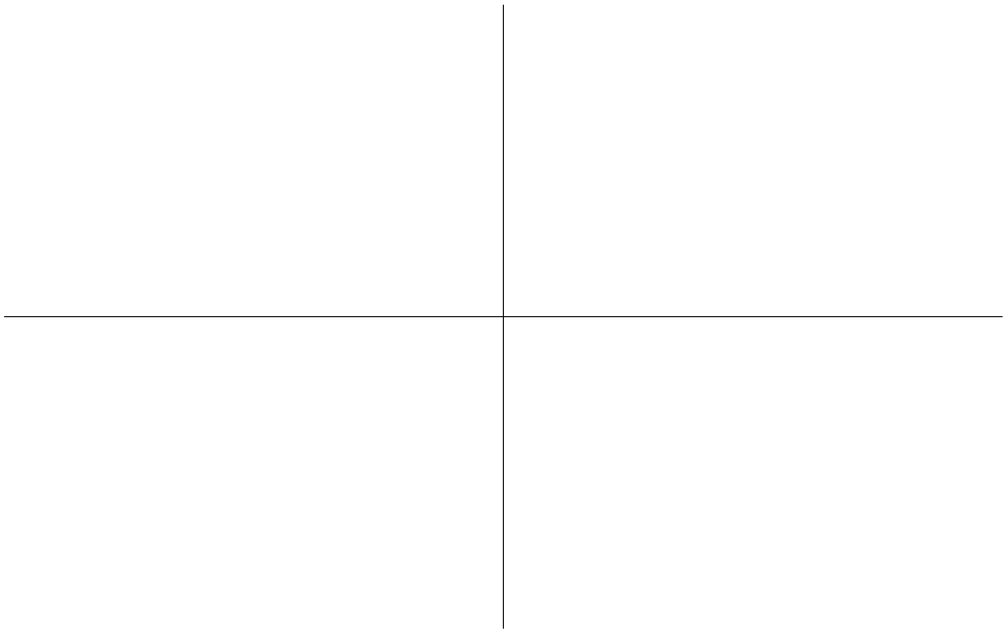
   (a) Does $r(x)$ intercept the x-axis? Where?

   (b) Does it have any vertical asymptotes? Where?

   (c) Where is $r(x)$ positive? Negative?

   (d) What happens as $x \to \infty$? What about as $x \to -\infty$?

(e) You should now have enough information to sketch a graph of $r(x)$. Do so on the axes below. Be sure to label all intercepts and asymptotes *before you start drawing the graph!*



A Helpful Method

Note that the sign (positive or negative) of a rational function can change only when it hits a zero or a vertical asymptote. This can help make sketching such functions much easier.

3. Consider the function $r(x) = \frac{(x-1)(x+2)}{(2x-1)(x+3)}$.

(a) $r(x)$ has zeros at $x =$ _____ and $x =$ _____.

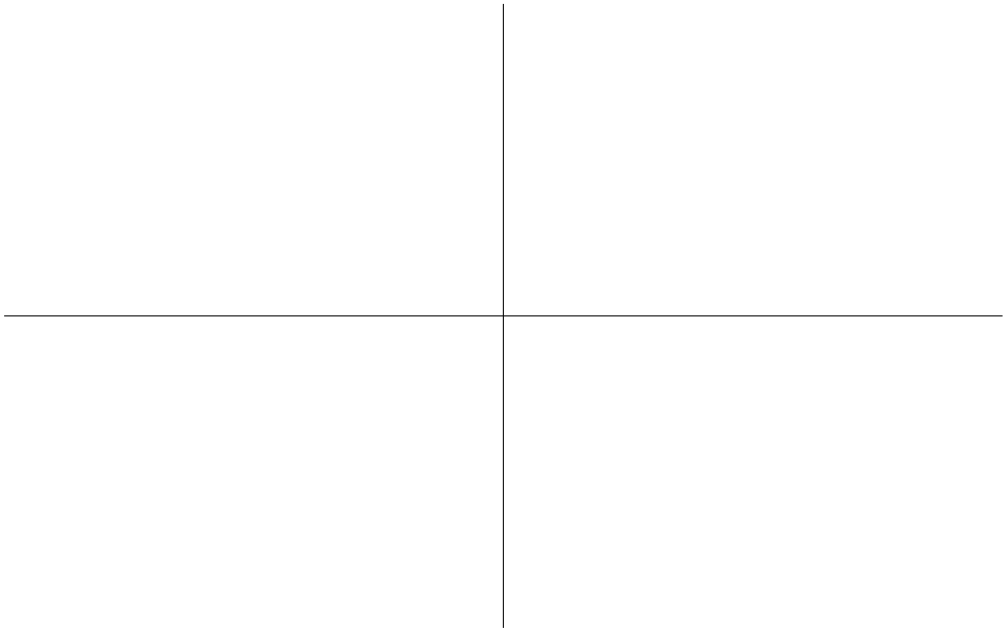
(b) $r(x)$ has vertical asymptotes at $x =$ _____ and $x =$ _____.

(c) Fill in the following table:

	$(x-1)$	$(x+2)$	$(2x-1)$	$(x+3)$	Total Sign
$x <$ ___					
___ $< x <$ ___					
___ $< x <$ ___					
___ $< x <$ ___					
___ $< x$					

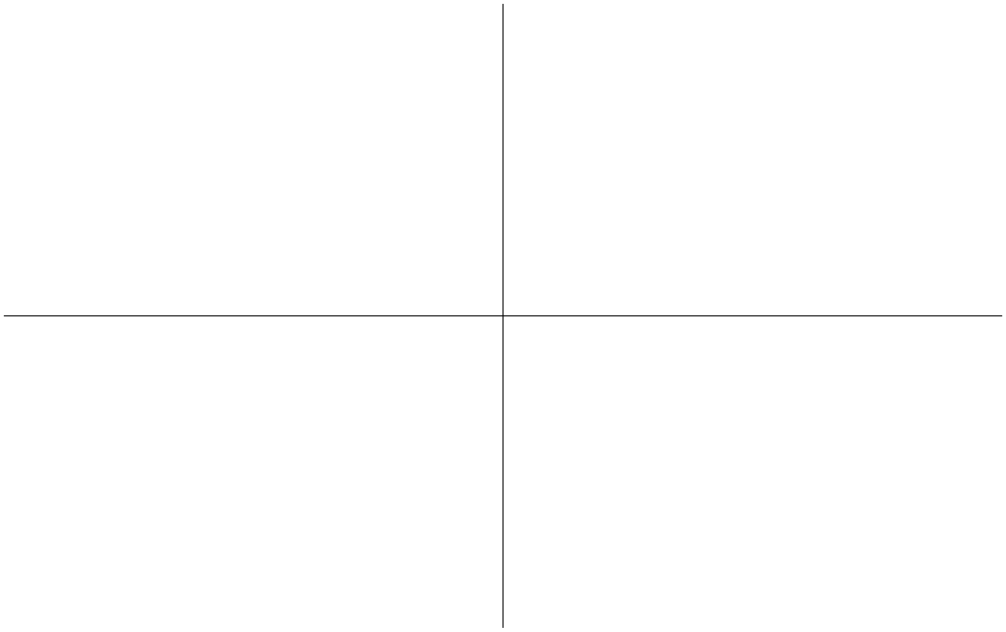
(d) What happens as $x \to \infty$? What about as $x \to -\infty$?

(e) Now draw the graph of $r(x)$.



4. By repeating the steps from the previous question, sketch the graph of function

$$f(x) = \frac{(x+1)(x+2)(x+3)}{x(x-1)^2}.$$

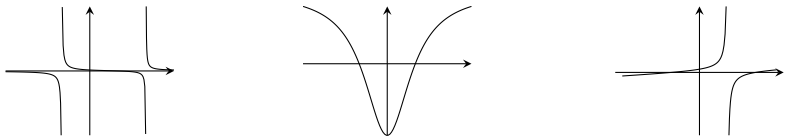


Extra Homework Problems

1. Match each of the following rational functions with the correct graph.

$$\frac{x^2-1}{x^2+1} \qquad \frac{x-1}{(x+1)(x-2)} \qquad \frac{(x+1)(x-2)}{x-1}$$

(**A**) (**B**) (**C**)



_____ _____ _____

2. Find all of the intercepts and asymptotes of the following functions.

   (a) $r(x) = \dfrac{x}{x^2-4}$

   (b) $r(x) = \dfrac{x+3}{x-3}$

   (c) $r(x) = \dfrac{(x+1)(x-1)}{x+2}$

3. Sketch the graph of $r(x) = \dfrac{(2x+2)(x-2)}{(x-1)(x+2)}$. Be sure to include and label all of the intercepts and asymptotes.