Preliminaries

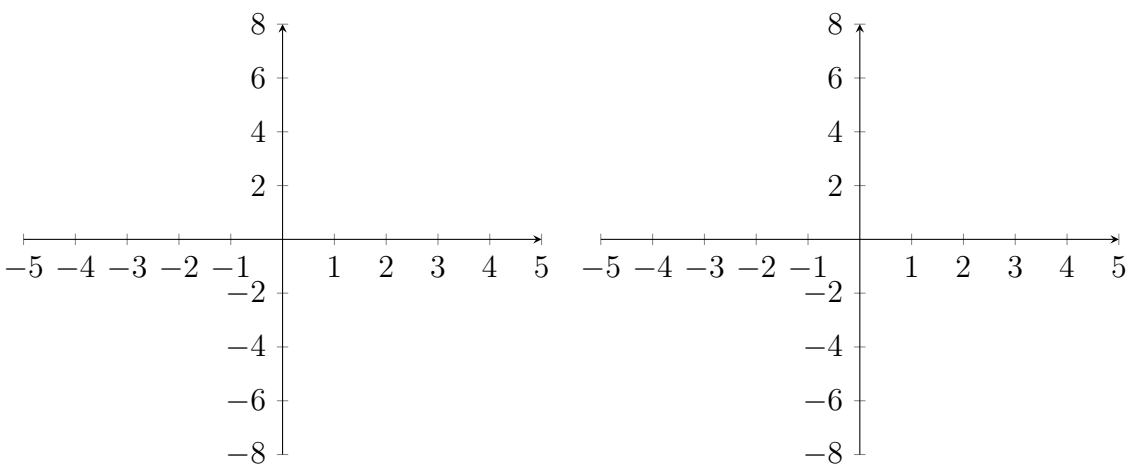
Power Functions

A *power function* is a function of the form

$$f(x) = kx^p,$$

where k and p are constants.

Question On the left-hand axes below, draw the graphs of $f(x) = x$, $f(x) = x^3$ and $f(x) = x^5$. On the right-hand axes, draw the graphs of $f(x) = x^2$ and $f(x) = x^4$. Take care to show the relative positions of the graphs and where they intercept. Then fill in the table below



	$\lim_{x\to-\infty} x^n$	$\lim_{x\to\infty} x^n$
n odd		
n even		

Polynomials

A *polynomial* is a sum of power functions with positive integer (whole number) exponents:

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0,$$

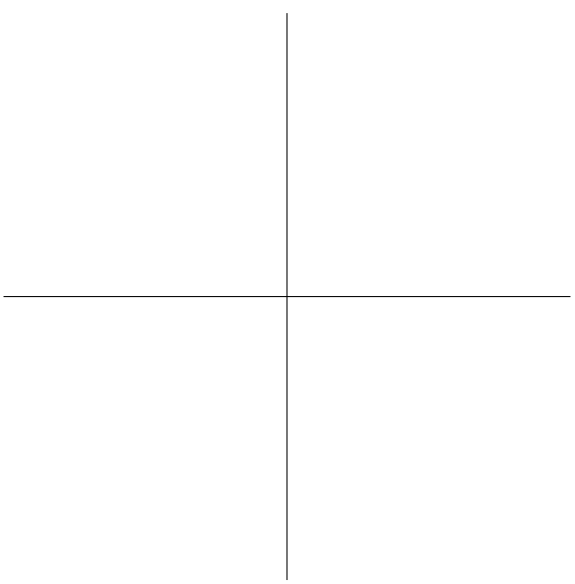
where the a_i are constants, and $a_n \neq 0$. The number n is called the *degree* of the polynomial.

- A second-degree polynomial ($n = 2$) is called a *quadratic*;
- A third-degree polynomial ($n = 3$) is called a *cubic*;
- A fourth-degree polynomial ($n = 4$) is called a *quartic*;
- A fifth-degree polynomial ($n = 5$) is called a *quintic*;

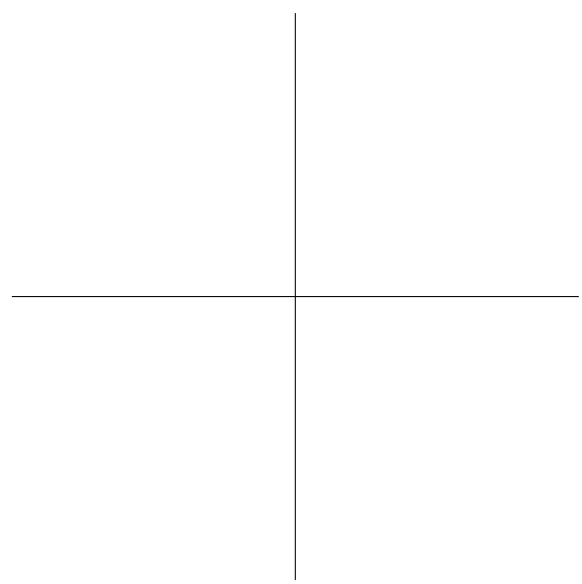
Question For this question, we will use the online software *GeoGebra*. Your teacher will show you how to open it and the basics of using it.

Use GeoGebra to plot the polynomials below. On the axes below, draw each of them from the graphs on your screen. Move each graph around and zoom in and out to make sure you see all its features! Fill in the blanks under each graph.

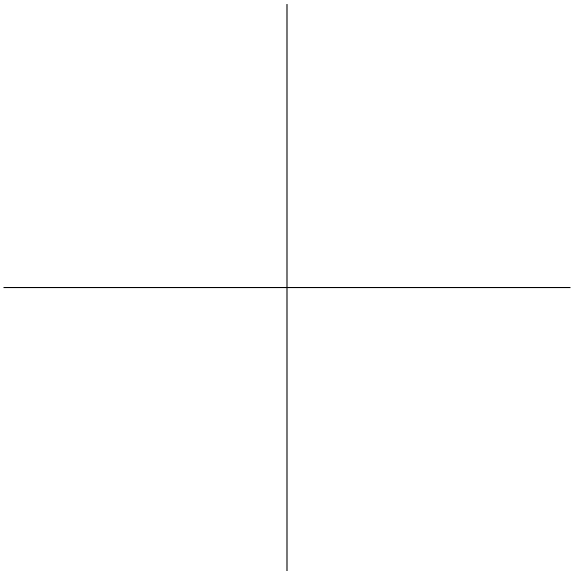
- $p(x) = x^2 - x + 2$
- $p(x) = x^3 + x^2 - 2x - 6$
- $p(x) = x^4 - x^3 - 7x^2 - 2x - 6$
- $p(x) = x^5 - x^4 - 5x^3 + 2x - 6$



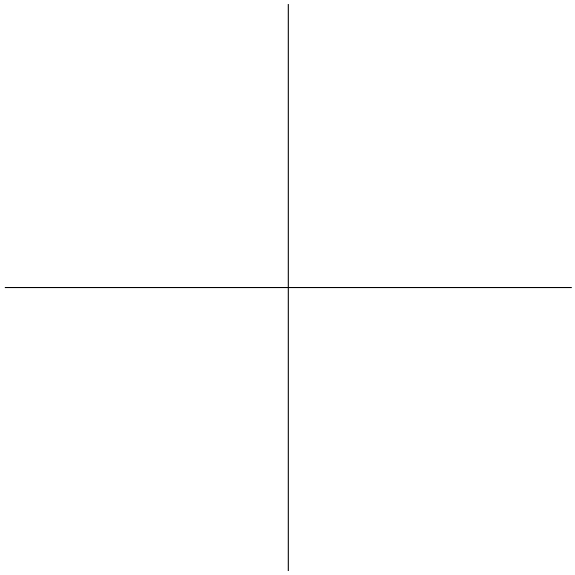
Degree: _____. # of turning points: _____



Degree: _____; # of turning points: _____



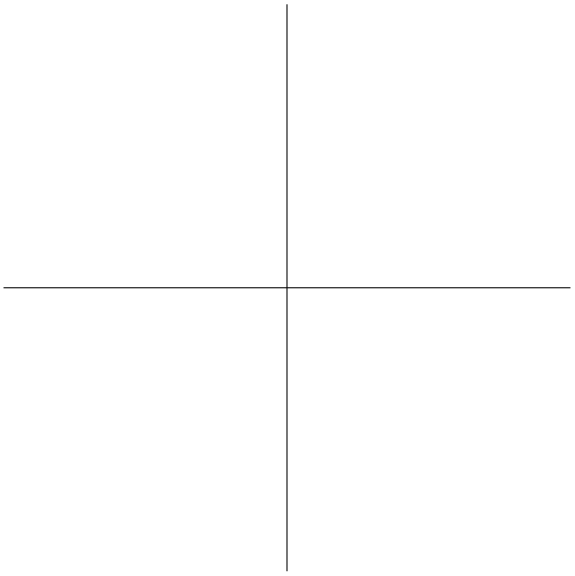
Degree: _____. # of turning points: _____



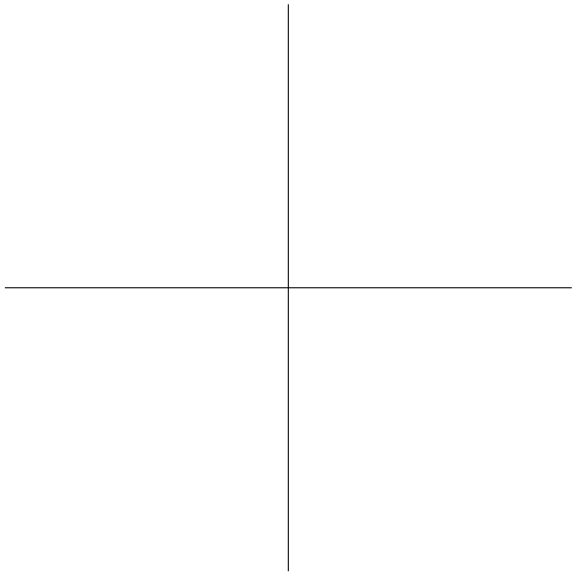
Degree: _____; # of turning points: _____

It seems that if a polynomial has degree n, then it has _____ turning points.

But, this is not always true! Draw a polynomial of degree three that has no turning points, and a polynomial of degree four that has only one. Be sure to label each one with its formula. (Hint: See page 1)



$f(x) =$ _____



$f(x) =$ _____

We will see later in the semester that the following is true:

A polynomial of degree n can have *at most* $n-1$ turning points.

Drawing Factored Polynomials

If we have a polynomial that is *factored* into linear factors, we can immediately see its x intercepts! We can find the y-intercept by multiplying out the constants.

For example, the polynomial

$$f(x) = (x-1)(x+2)(x-3)$$

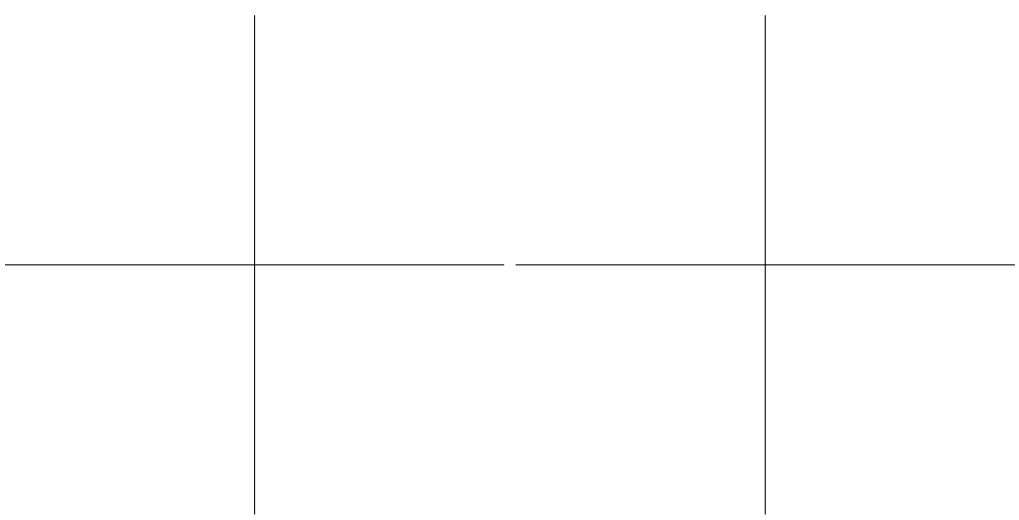
has x intercepts at $x = 1, -2$, and 3, and a y-intercept at $y = -1 \times 2 \times -3 = 6$.

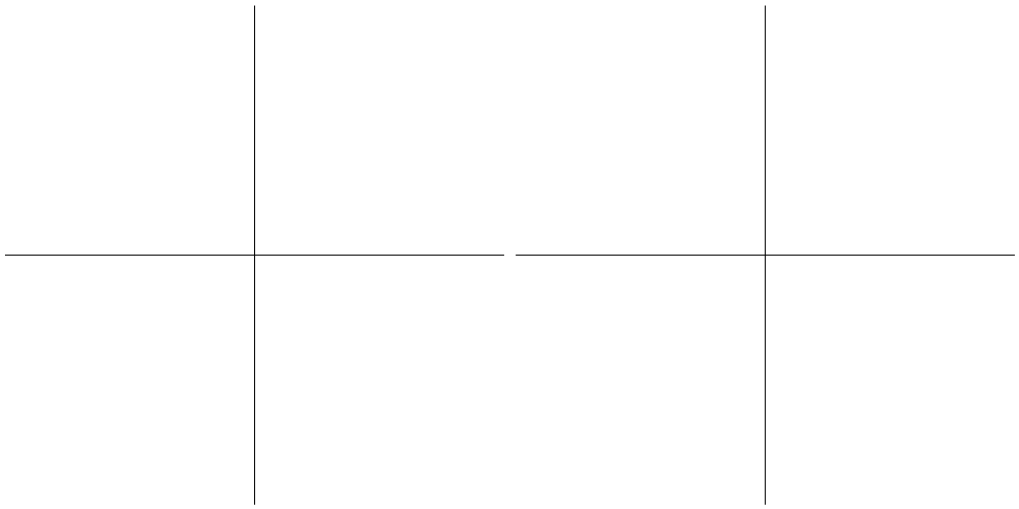
- If the degree of a polynomial is *even*, then:
  - its tails go in the *same* direction: either both to ∞ or both to $-\infty$.
  - if the coefficient of the top term is *positive*, both tails go to ∞.
  - if the coefficient of the top term is *negative*, both tails go to $-\infty$
- If the degree of a polynomial is *odd*, then:
  - its tails go in *opposite* directions: one goes to ∞ and the other to $-\infty$.
  - if the coefficient of the top term is *positive*, the tail on the left goes to $-\infty$, and the tail on the right goes to ∞.
  - if the coefficient of the top term is *negative*, the tail on the left goes to ∞, and the tail on the right goes to $-\infty$.

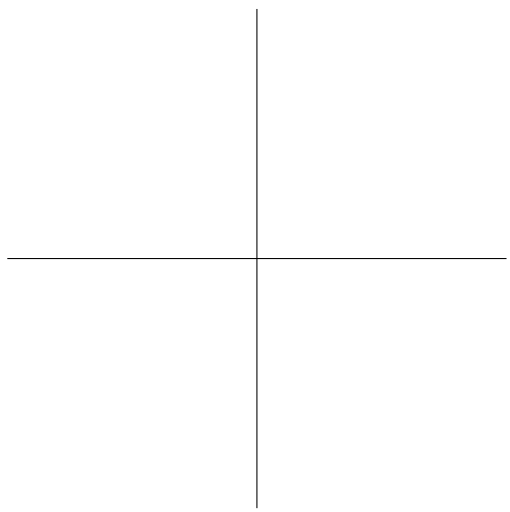
This is enough information to draw the graphs of factored polynomials!

1. Draw graphs of the following polynomials. This time, make sure you label all the x-
□Regrade and y-intercepts precisely:

(a) $y = x(x-1)(x+2)$.
(b) $y = (x+1)(x-1)^2$.
(c) $y = -x^2(x-1)$.
(d) $y = \frac{1}{8}(x+1)(x-2)^3$
(e) $y = -\frac{1}{2}(x-1)(x+1)^2(x-2)^2$

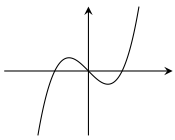
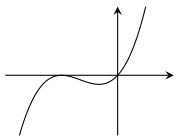
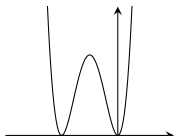
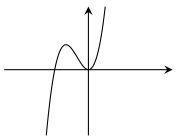






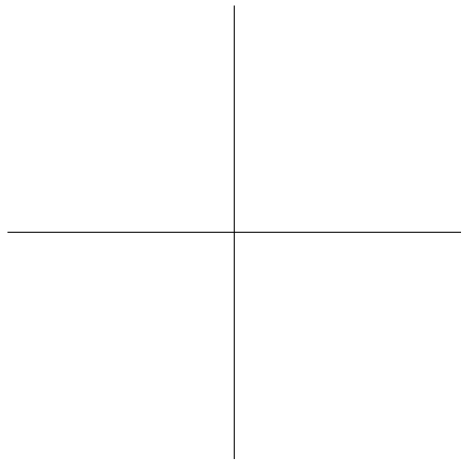
2. Match each of the following polynomials with the correct graph.

☐Regrade

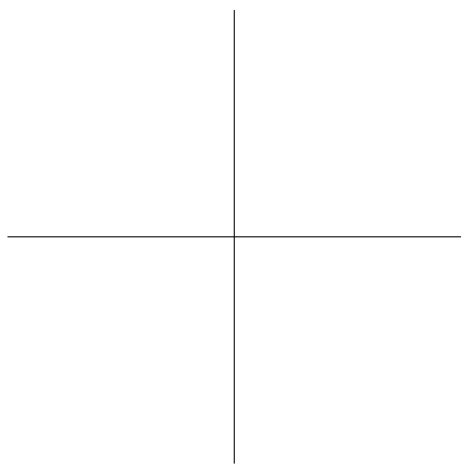
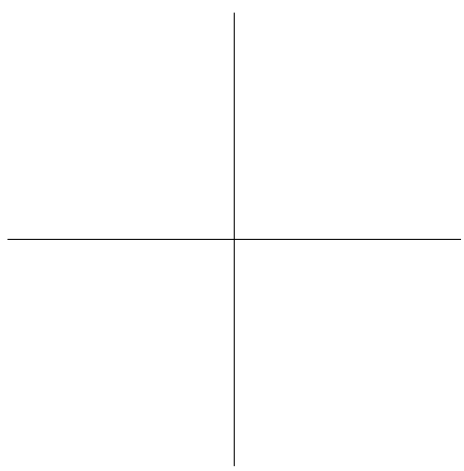
$x^2(x+1)$	$x(x+1)(x-1)$	$x(x+1)^2$	$x^2(x+1)^2$
(A)	**(B)**	**(C)**	**(D)**
			
_____	_____	_____	_____

3. By first factoring, sketch the graphs of the following polynomials. Be sure to label all ☐Regrade of the intercepts.

(a) $p(x) = x^2 - 1$



(b) $p(x) = x^3 - x$



(c) $p(x) = -\frac{1}{2}(x^4 - 2x^3 + x^2)$