

Power Functions

A *power function* is a function of the form

$$f(x) = kx^p,$$

where k and p are constants.

Question

1. (a) On the left-hand axes below, draw the graphs of $f(x) = x$, $f(x) = x^3$ and $f(x) = x^5$. On the right-hand axes, draw the graphs of $f(x) = x^2$ and $f(x) = x^4$.



- (b) Fill in the following table:

	$\lim_{x \rightarrow -\infty} x^n$	$\lim_{x \rightarrow \infty} x^n$
n odd		
n even		

Polynomials

A *polynomial* is a sum of power functions with positive integer (whole number) exponents:

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0,$$

where the a_i are constants, and $a_n \neq 0$. The number n is called the *degree* of the polynomial.

- A second-degree polynomial ($n = 2$) is called a *quadratic*;
- A third-degree polynomial ($n = 3$) is called a *cubic*;
- A fourth-degree polynomial ($n = 4$) is called a *quartic*;
- A fifth-degree polynomial ($n = 5$) is called a *quintic*;

Questions For this question, we will use the online software *GeoGebra*. Your teacher will show you how to open it and the basics of using it.

2. Use GeoGebra to plot the polynomials below. On the axes below, draw each of them from the graphs on your screen. Move each graph around and zoom in and out to make sure you see all its features!

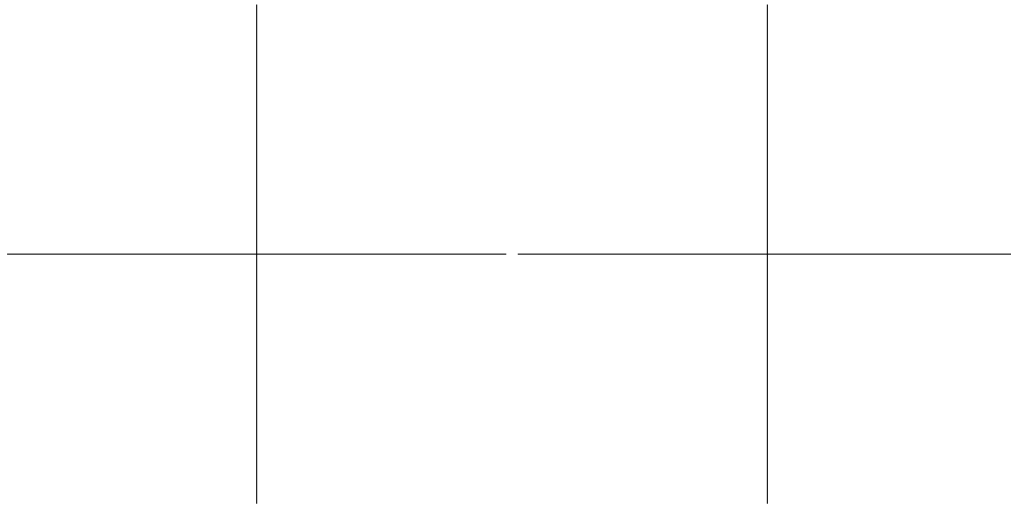
(a) $p(x) = x^2 - x + 2$

(b) $p(x) = x^3 + x^2 - 2x - 6$

(c) $p(x) = x^4 - x^3 - 7x^2 - 2x - 6$

(d) $p(x) = x^5 - x^4 - 5x^3 + 2x - 6$





3. Draw graphs of the following polynomials. This time, make sure you label all the x - and y -intercepts precisely:

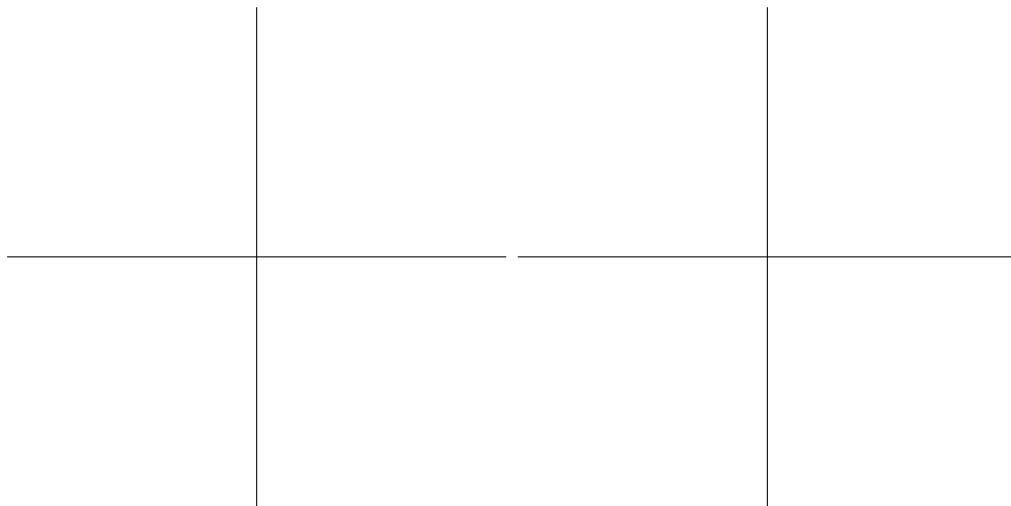
(a) $y = x(x - 1)(x + 2)$.

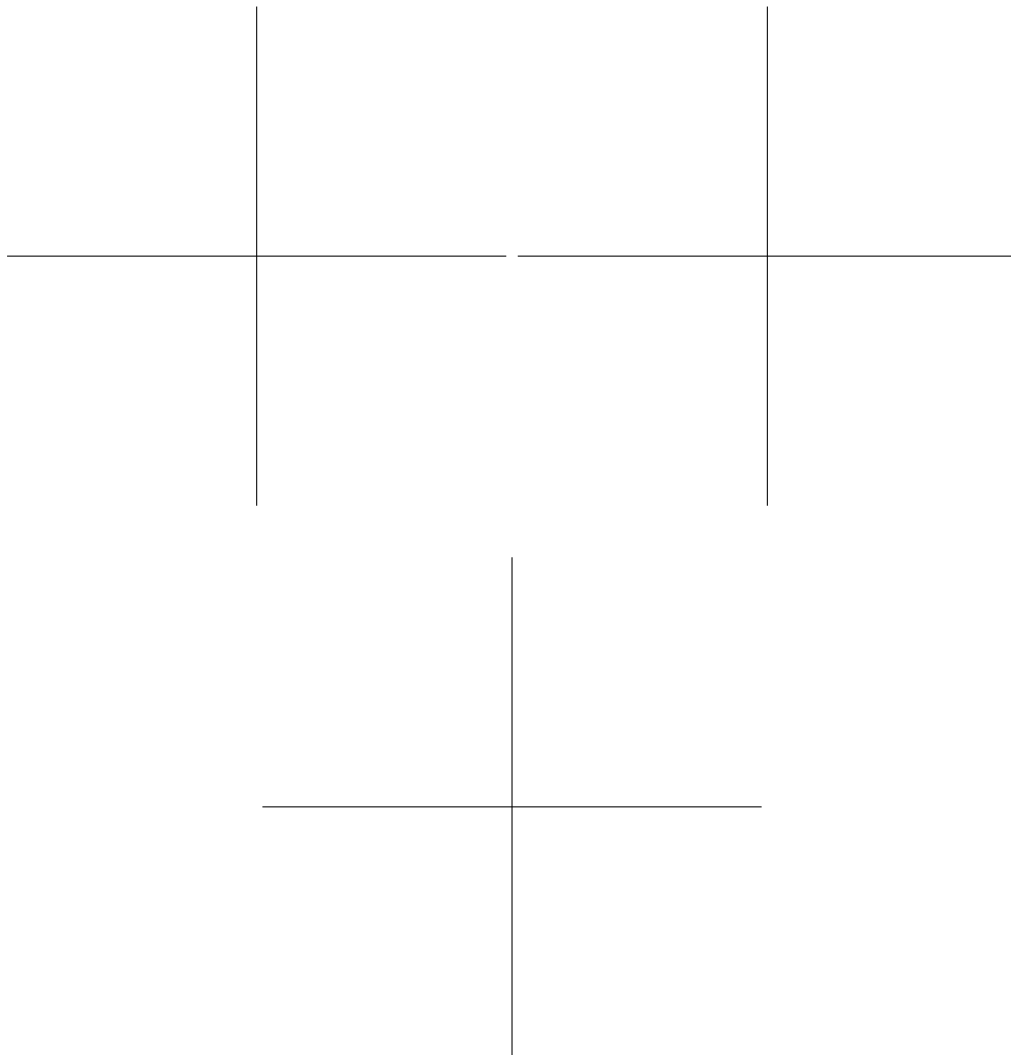
(b) $y = (x + 1)(x - 1)^2$.

(c) $y = -x^2(x - 1)$.

(d) $y = \frac{1}{8}(x + 1)(x - 2)^3$

(e) $y = -\frac{1}{2}(x - 1)(x + 1)^2(x - 2)^2$





4. If the degree of a polynomial $p(x)$ is n does the graph of $p(x)$ always ‘turn around’ $n - 1$ times? Explain!

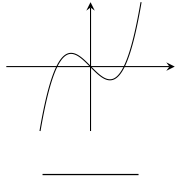
5. What changes about the behavior of the ‘tails’ of the graph of a polynomial if its leading coefficient is negative? Compare question 2(c) and 2(e) to questions 1(b) and 1(d), for example.

Extra Homework Problems

1. Match each of the following polynomials with the correct graph.

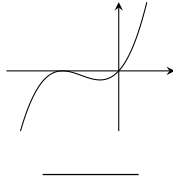
$$x^2(x+1)$$

(A)



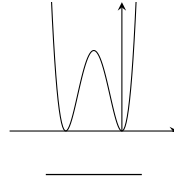
$$x(x+1)(x-1)$$

(B)



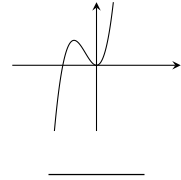
$$x(x+1)^2$$

(C)



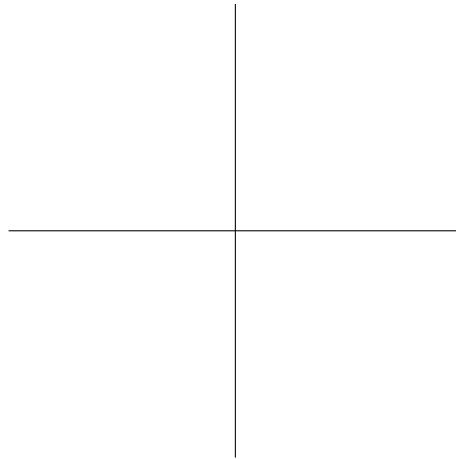
$$x^2(x+1)^2$$

(D)

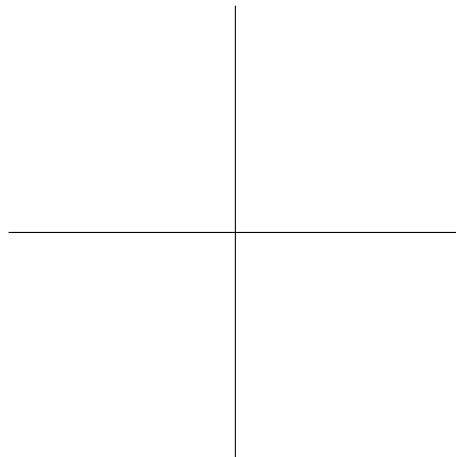


2. Sketch the graphs of the following polynomials. Be sure to label all of the intercepts.

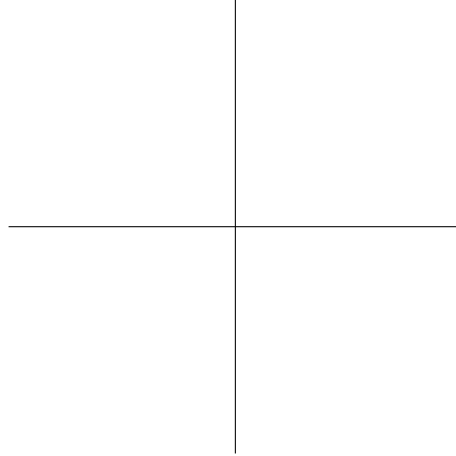
(a) $p(x) = x^2 - 1$



(b) $p(x) = x^3 - x$



(c) $p(x) = x^4 - x^2$



(d) $p(x) = -\frac{1}{2}(x - 2)^2(x + 1)^3(x - 5)$

