Review

Question

1. Assuming x and y are functions of t, compute the following derivatives. Note: these are implicit derivatives!

   (a) $\frac{d}{dt}\sqrt{x^2+y^2}$

   (b) $\frac{d}{dt}x^2y$

Related Rates

Questions

2. You are blowing up a spherical balloon at a rate of $50\text{cm}^3/\text{min}$.

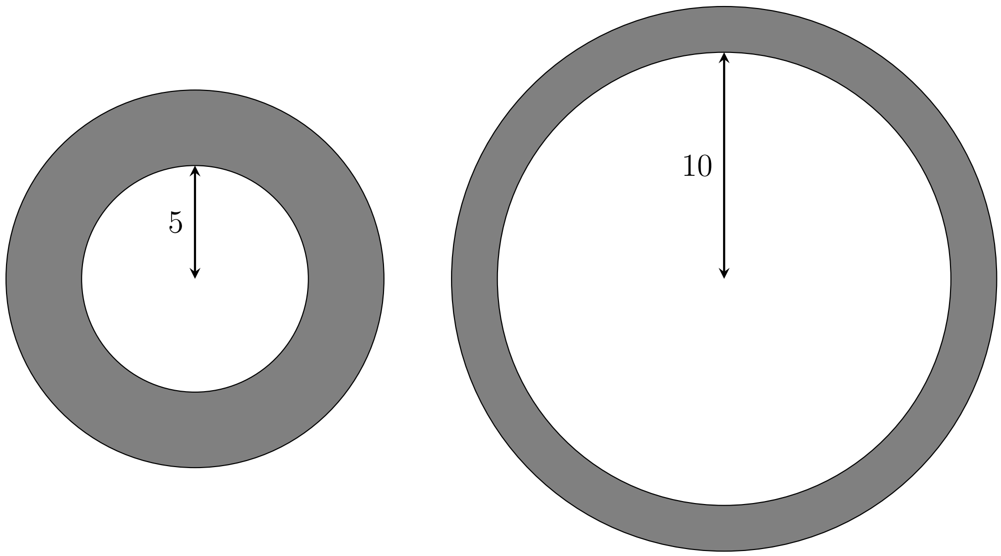
   (a) What is the volume V of the balloon when its radius is r cm?

   (b) Since the radius is changing, it is a function of time. Differentiate your answer with respect to time, t.

   (c) Use the fact that the rate of change of volume is $50\text{cm}^3/\text{min}$ and the previous answer to find the rate of change of the radius when the radius is 5cm.

   (d) What about when the radius is 10cm?

(e) Is the radius changing faster when the balloon has radius 5cm or radius 10cm? Use the picture to explain why this makes sense. Note that the shaded areas are the same.



3. Suppose two cars leave an intersection simultaneously. One travels north at 30 mi/hr, while the other travels east at 40 mi/hr. How fast is the distance between the two changing after one minute?

4. The radius r and height h of a cone are related to the cone's volume V by the formula $V = (1/3)\pi r^2 h$.

   (a) How is $\dfrac{dV}{dt}$ related to $\dfrac{dh}{dt}$ if r is constant?

   (b) How is $\dfrac{dV}{dt}$ related to $\dfrac{dr}{dt}$ if h is constant?

   (c) How is $\dfrac{dV}{dt}$ related to $\dfrac{dr}{dt}$ and $\dfrac{dh}{dt}$ if neither$r$ nor h is constant?

5. The length l of a rectangle is decreasing at the rate of 2 cm/s while the width w is increasing at the rate of 2 cm/s.

   (a) When $l = 12$ cm and $w = 5$ cm, what is the rate of change of the area of the rectangle? Is it increasing or decreasing?

   (b) When $l = 12$ cm and $w = 5$ cm, what is the rate of change of the perimeter of the rectangle? Is it increasing or decreasing?

6. The ideal gas law says _________________________. If the quantity of gas and temperature remain fixed, this becomes Boyle's Law. Then, the pressure P and the volume V satisfy $PV = k$ for some constant k. Suppose a certain quantity of gas occupies 10 cubic centimeters at a pressure of 2 atmospheres. If the pressure is increasing at a rate of 0.05 atmospheres per minute when the pressure is 2 atmospheres, find the rate at which the volume is changing at that moment. What are the units of your answer?

7. Suppose that we steal the world's largest marshmallow[1]. It is a cylinder of volume 7300 cubic feet with a base radius of 12.5 feet. In order to make the world's largest s'more out of it, we heat it and find that the radius is changing at a rate of .6 feet per minute and the height is changing at a rate at a rate of .4 feet per minute. How fast is the volume changing just as we start to heat the marshmallow?

[1] Found in Noble County, Indiana and weighing 2300 pounds

8. A manufacturer of tennis balls decides to increase production by 30 cans each day. The manufacturer has determined that the total revenue R (in dollars) from the sale of x cans of tennis balls in a day is approximately given by the function $R(x) = 2.14x - 0.0003x^2$. Determine the rate of change of revenue with respect to time when the daily production level is 1200 cans. (Assume all cans are sold.)

9. Suppose that a drop of mist is a perfect sphere and that, through condensation, the drop picks up moisture at a rate proportional to its surface area. Show that under these circumstances the radius of the drop increases at a constant rate.

10. Water is draining at the rate of 50 m^3/min from a shallow concrete conical reservoir (vertex down, i.e., the ice cream cone is right-side up). The conical reservoir has a top radius of 45 meters and a height of 6 meters.

    (a) How fast is the water level falling when the water is 5 meters deep? (Convert your answer to cm/min.)

    (b) How fast is the radius of the water's surface changing then? (Again, convert your answer to cm/min.)