What Is Optimization?

In many applications, we want to *optimize* some quantity subject to constraints. For example, we might want to build a box with a given volume using minimal materials (aka minimal surface area). We might want to use a nicer (more expensive) material for, say, a decorative top for the box, and we want to choose dimensions for it that minimize total cost.

These are questions about global maxima and minima, which we studied on the last worksheet. We will need to take a written description, translate it to a function, then apply the same ideas from then. Along the way, we will often need to eliminate a variable using a constraint.

Example Suppose that we have a function of *two* variables: $C(x, y) = x^2y$. We can't find its max and min, since we have no idea how to take derivatives when two variables are involved. Suppose, though, that we also know that $2x + y = 9$. Use this to write $C(x, y)$ as a function of x alone. Then find the global max of C on $0 \leq x \leq 4.5$. What is the value of y there? What is the value of C?

Example You want to make a rectangular box with square base and open top that will have a volume of 270 cubic inches. The material for the bottom of the box costs 48 cents per square inch. The material for the sides costs 3 cents per square inch. Find the dimensions of the box that will minimize your total cost. What is the total cost per box?

Example Your rectangular box project from the previous question was so successful that you want to make a series of boxes with the same construction, but with varying volumes. Determine the dimensions that minimize the total cost in terms of V, the volume of the box in cubic inches. Then find the cost in terms of V. Use your answer to check the previous example.

Example Evilcorp, the company you work for, wants to distribute millions of copies of a page of their advertising propaganda. The page is to contain 24 square inches of printed material, with margins of 1.5 inches at the top and bottom of the page, and 1 inch at the sides. Since paper is expensive, Evilcorp wants to minimize the amount of paper needed to print such a page. Find the dimensions (length and width) of the page that will minimize the total area.

Problems

1. For her birthday, your mother wants a painted jewelry box with a square base and
□Regrade a volume of 96 cubic inches. If the top of the box costs 6 cents per square inch to paint, and painting the sides and base of the box costs 3 cents per square inch, what dimensions of the box will minimize the total cost of painting?

2. Evilcorp tells you that the margin sizes they told you before are wrong (see the last worked example), but that they haven't figured out the new ones yet. Find the overall dimensions of the page that minimize the total area of paper, in terms of T and S, the top/bottom and left/right margin lengths. Use your answer to verify the worked example.

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3. Waste Management, Inc. makes small, heavy-duty cans (which are cylindrical with closed top and bottom, of course) out of two kinds of material. The material used to make the top and bottom of the can costs 6 cents per square inch, and the material used to make the curved surface of the can costs 3 cents per square inch. The company is willing to pay \$1.44 for the material used in each can, but wants the can to hold as much waste as possible. If r is the radius and h is the height of the can, find the value of r that will maximize the amount of waste the can will be able to hold.

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4. A rectangular box with square base and open top is constructed from two types of
□Regrade material. The material used to make the bottom of the box costs 10 cents per square inch, and the material used to make the rest of the box costs 6 cents per square inch. The total cost of the box is \$3.00. If s is one side of the base of the box and h is the height, find the value of s that maximizes the volume V.

5. The Transcam company has just landed a big contract for steel wool. Because steel
□Regrade wool is relatively light, the shipping department wants to pack as much as possible into each box sent out to cut down on handling charges. The trucking company allows boxes to have a combined length and girth (the distance measured around the box perpendicular to its length) of no more than 108 inches. If the shipping department uses boxes with square ends, what box dimensions will maximize the volume the box can hold?

6. Suppose that Transcam (see the previous problem) decides to ship its steel wool in cylindrical containers instead of boxes. □Regrade

(a) What height and radius will maximize the volume of the cylindrical containers?

(b) Which is the better choice for Transcam, the box or the cylindrical container? Explain.