Review from Earlier This Week

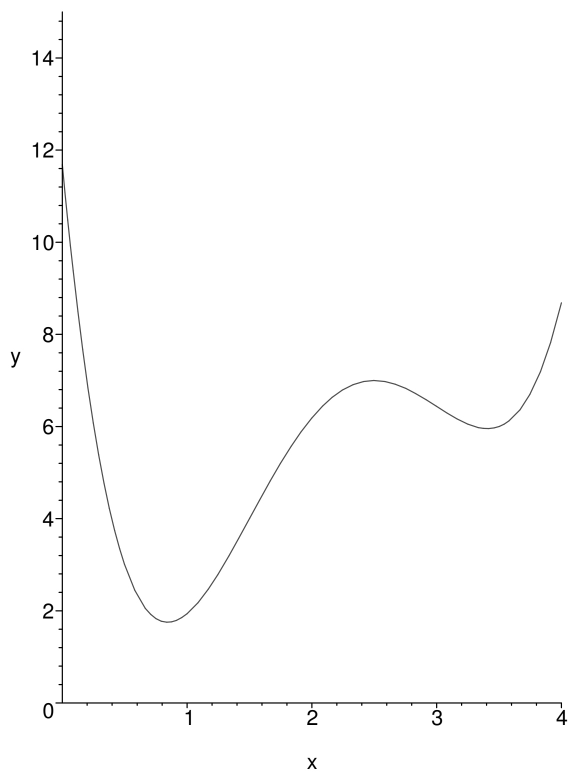
- **First Derivative Test:**
  Let $f(x)$ be a function and let a be in the domain of f. Then:
  - If ______________________________, then f has a local max at a.
  - If ______________________________, then f has a local min at a.
- **Second Derivative Test:**

Local vs. Global Extrema

During the week, we talked about local max and mins of functions. Today we are going to look at global max and mins. A **global maximum** of a function f is the greatest value of f over a specified domain. A **global minimum** is the least value of f over the domain.

Example

1. Consider the function below.

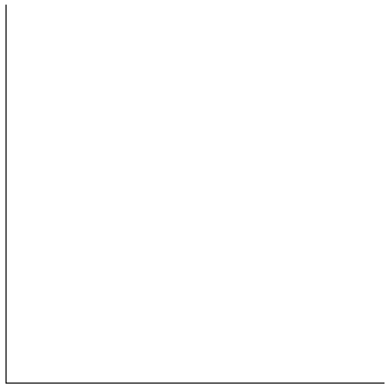


   (a) Identify the local and global mins and maxes.

   (b) Using the graph above to guide you, what are the possible locations for the global maximums and minimums?

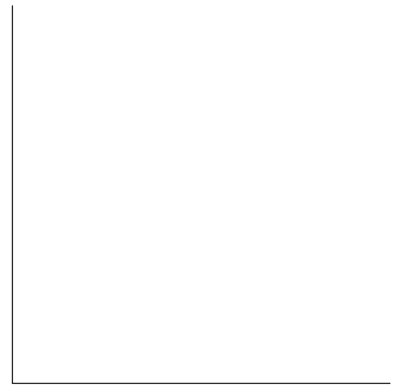
Question

2. Draw the graph of $g(x) = \frac{1}{x}$ on the axes below.



Does g have a global maximum over the interval $0 < x \leq 1$? Explain.

3. Draw the graph of $h(x) = x^2$ on the axes below.



Does h have a global maximum over the interval $0 \leq x < \infty$? Explain.

Extreme Value Theorem: If f is continuous on a closed finite interval $a \leq x \leq b$, then $f(x)$ has both a global maximum and a global minimum on that interval.

Questions

4. Why do $g(x)$ and $h(x)$ fail this theorem?

5. Explain why $f(x) = x$ does not have a global min on $-2 < x < 2$, but $g(x) = x^2$ does.

6. Find the global max and min of $f(x) = x(x - 1)$ on the interval $0 \leq x \leq 3$.

Procedure for finding the global max and min of a function f

- Find the critical points of f.
- Find the values of f at the critical points and at the endpoints.
- Determine the global max and min from these values.

Continuous Functions on Finite Intervals

Questions

7. For the following functions, find the global maximum and minimum on the specified domain, if they exist.

   (a) $f(x) = x^3 - 7x + 6$ on the interval $-4 \leq x \leq 2$

   (b) $g(x) = \ln(1 + x^2)$ for $-1 \leq x \leq 2$

   (c) $h(x) = \frac{x^3}{3} - x^2 + x$ on $0 < x \leq 2$

When Things Get a Little Harder...

Questions

8. Find the global max and min of $f(t) = te^{-t}$ for $t \geq 0$. (Hint: careful here, as the domain is neither closed nor finite)

9. Find the global max and global min of the function $p(x) = \ln x^2 - (x-1)^2 + 8$ on $-3 \leq x \leq 6$. Where (i.e. at what values of x) do the global max and global min occur? (Hint: careful! What happens when $x = 0$?)