From Way Back When....

- The derivative of a function $f(x)$ at *a point* a is

$$f'(a) = \_\_\_\_\_.$$

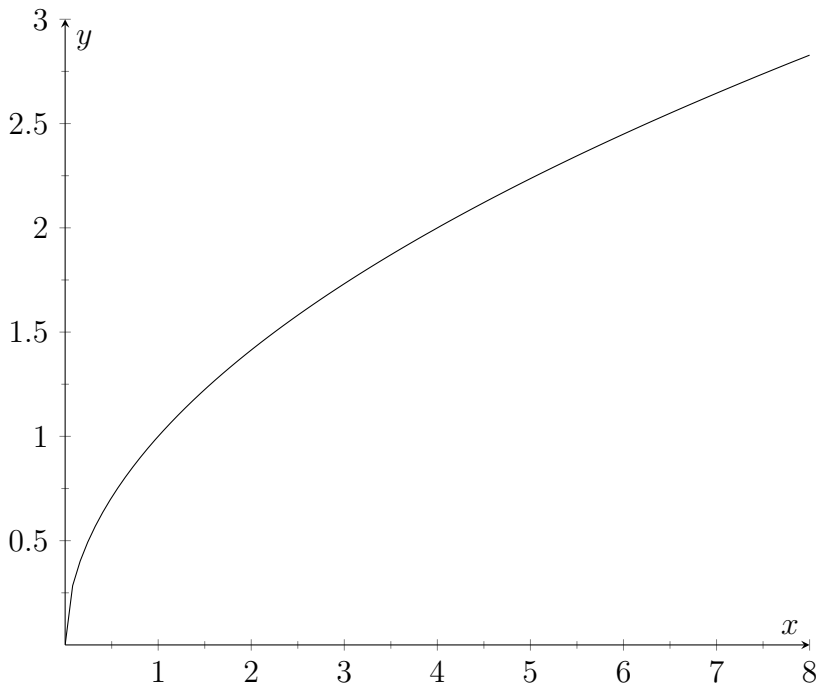
- The derivative *function* of a function $f(x)$ is

$$f'(x) = \_\_\_\_\_.$$

Tangent Lines and Linear Approximations

Question Let $f(x) = \sqrt{x}$.

1. Draw the graph of $f(x)$ for x in the domain $[0, 8]$ on the axes below:



2. Find the equation for the tangent line to $f(x)$ at the point $x = 4$ and draw it on top of the graph of $f(x)$. Try to be as precise as possible.

3. Draw a very small box around the point $(4, 2)$.

   (a) What do you notice? Can you explain your observation?

   (b) Suppose you only have a very simple calculator with $+$, $-$, $\times$, and $\div$ operators. Use your work above to find a good approximation of $\sqrt{4.1}$.

   (c) Use your calculator to get the value of $\sqrt{4.1}$ to six decimal places. How far off was your approximation above?

4. Let's see what happens if we try to use the same idea to find $\sqrt{6}$:

   (a) By looking at the graphs you draw on the previous page, do you expect that your approximation will be as good as your approximation of $\sqrt{4.1}$? Why or why not?

   (b) Use the same method as above to approximate $\sqrt{6}$.

   (c) Now use your calculator again to get the value of $\sqrt{6}$ to four decimal places. How far off was your approximation?

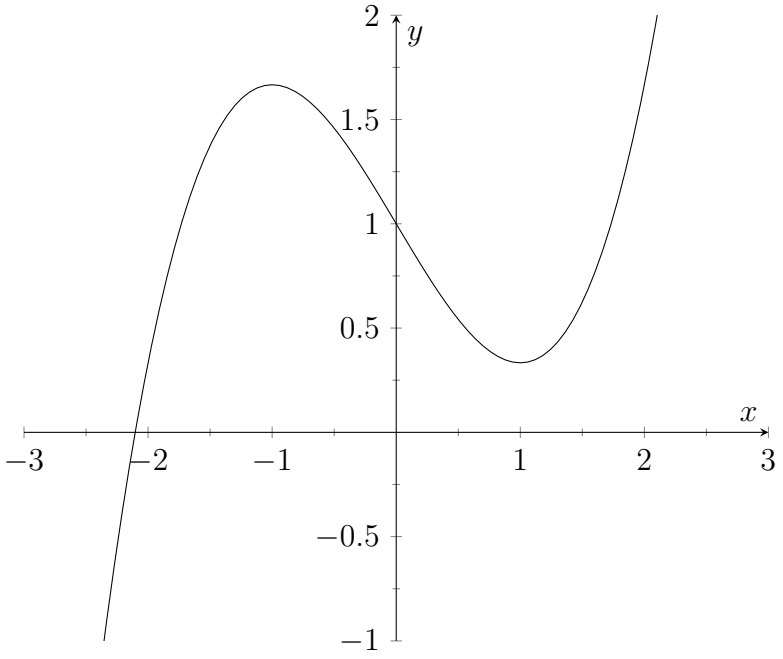
Definition When we refer to 'the linear approximation of $f(x)$ at a point $x = a$', we mean the tangent line to the graph of $f(x)$ at that point.

Questions

5. Let $y = \ln(x)$. Find the linear approximation at $x = 1$. Then draw the curve and the tangent line, and use the latter to estimate $\ln(1.1)$. How far off is your approximation? Do the same for $\ln(1.5)$. Are your estimates overestimates of the true values of $\ln(x)$ at 1.1 and 1.5 or underestimates?

6. Use linear approximations to estimate $e^{0.1}$ and $e^{0.5}$. (Hint: where would it make sense to base your linear approximation?) Are your estimates of the values of e^x at 0.1 and 0.5 overestimates or underestimates? Draw a graph (and a tangent line) to explain your answers.

7. Consider the function graphed below



(a) Draw the tangent lines to curve at the point $x = -1.5$ and $x = 1.25$.

(b) At $x = -1.5$, the linear approximation will overestimate/underestimate the value of the function at nearby points. (Cross out the wrong answer.)

(c) At $x = 1.25$, the linear approximation will overestimate/underestimate the value of the function at nearby points. (Cross out the wrong answer.)

(d) Under what circumstances are (b) and (c) true? Why?

8. Suppose that $f(x)$ is differentiable function, that $f(4) = 3$ and $f'(4) = 6$.

(a) Write down the linear approximation to $f(x)$ at $x = 4$.

(b) Use your approximation to estimate $f(3.9)$.

(c) If $f''(x) < 0$ on the interval $[3.9, 4]$, is your estimate an overestimate or an underestimate?