Preliminaries: From Times Gone By...

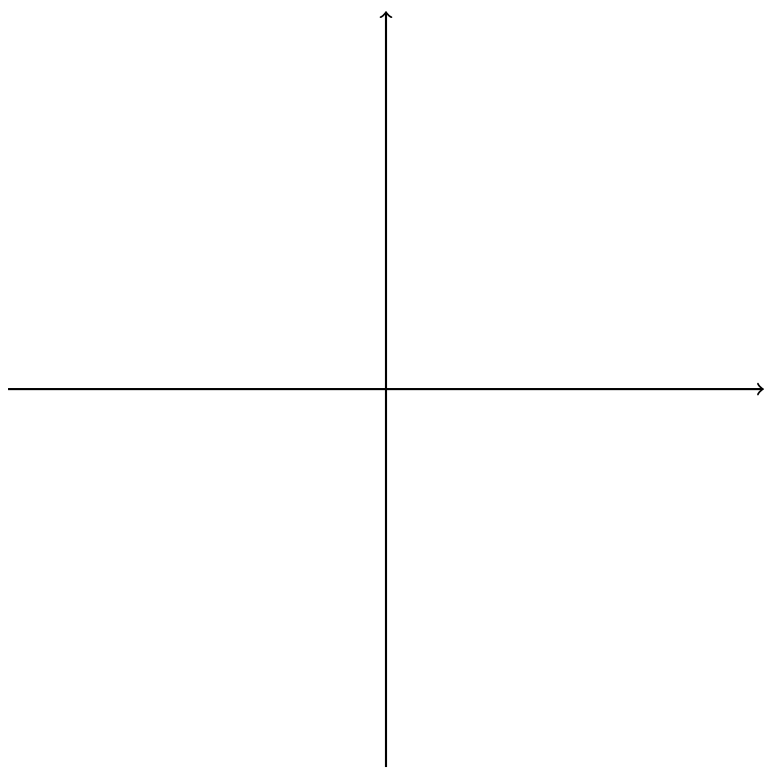
- If $f(x) = a^x$, then $f'(x) =$ _____.
- The inverse function of $g(x) = 10^x$ is $g^{-1}(x) =$ _____.
- The inverse function of $h(x) = e^x$ is $h^{-1}(x) =$ _____.
- The derivative of a composite function $f(g(x))$ is

$$\frac{d}{dx} f(g(x)) = \underline{\hspace{6cm}}.$$

Derivative of $\ln(x)$

Sketching

On the axes below, draw $f(x) = \ln(x)$, then sketch its derivative. Your answer should look like (part of) a curve you know pretty well....



Computing

Consider the equation $y = \ln(x)$.

- Solve this equation for x
- Use implicit differentiation to find $\frac{dy}{dx}$ in terms of y.
- Use the relationship from the first bullet point to find the derivative of $\ln(x)$ in terms of x only.

Derivative of $\log(x)$

Carry out a similar process to find the derivative of $y = \log(x)$.

Derivatives of Logarithmic Functions

The derivative of $f(x) = \ln(x)$ is

$$f'(x) = \frac{d}{dx}\ln(x) = \underline{\hspace{3cm}}$$

The derivative of $f(x) = \log(x)$ is

$$f'(x) = \frac{d}{dx}\log(x) = \underline{\hspace{3cm}}$$

Questions

1. Differentiate the following functions:

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(a) $f(x) = \ln(2x + 6)$

(b) $g(x) = \log(x^2)$

(c) $h(x) = \ln(x^3)$

(d) $y = 6x^2e^x \ln(x)$ (Hint: Write this as $y = (6x^2e^x)\ln(x)$ and use the product rule twice.)

2. Find the equation of the tangent line to the curve $f(x) = 6x \ln x$ at $x = 3$.

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3. Consider the expression $e^{\ln x}$.

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(a) What does the expression simplify to? Fill in the blank: $e^{\ln x} =$ _____.

(b) Take derivatives of each side of the equation you just wrote down to check you get the same answer.

(c) Carry out the same process for the expression $\ln(e^x)$

4. (a) Find $\frac{d}{dx}\ln(-x)$

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(b) What is the domain of $\ln(-x)$?

(c) We conclude that $\frac{1}{x}$ is the derivative of _____ when $x > 0$ and of _____ when $x < 0$. Given that

$$|x| = \begin{cases} x & x > 0 \\ -x & x < 0 \end{cases}$$

write this as one expression:

$$\frac{d}{dx}\underline{\hspace{3cm}} = \frac{1}{x}$$

for all $x \neq 0$.