Linear Functions

A function is *linear* if its slope at every point is the same.

Equivalently, any linear function can be written in the form

$$y = f(x) = \underline{\hspace{3cm}},$$

where m is the ____________ and c is the ____________.

Questions

1. Suppose $f(x)$ is a linear function. Find its equation given that $f(2) = 4$ and $f(4) = -2$.

2. Suppose $g(x)$ is a linear function. Find its equation given that its slope is 6 and that $g(5) = 9$.

Fill in the blank: The slope, m, of a linear function is $\frac{\Delta y}{\Delta x}$. If $\Delta x = 1$, then $\Delta y =$ ___.

This leads to the following:

An Important Characterization of the Slope of a Linear Function:

Slope is the amount the ____________ variable increases by every time the ____________ variable increases by ___.

Question

3. One group calculated in the Cancer Mortality lab that if I is the index of exposure, and M is cancer mortality (in cancer deaths per 20,000 people per year), then a good model for the data is
$$M = 11.2I + 112.$$
If a mayor of a town manages to decrease the index of exposure in his town, whose population is 115,000 people, by exactly 1.7 units, how many cancer deaths does the model predict will be prevented over the next five years?

Question

4. **The Apple Orchard** In a certain orchard, the following facts are known:

   - if 24 trees are planted per acre, each tree will yield 600 apples and
   - for each additional tree planted per acre, the yield decreases by 12 apples per tree.

   Let the number of trees per acre be x, and denote the number of apples per tree by A.

   (a) What is A if x is 24 trees per acre? Include units.

   (b) What information in the question tells you that A is a *linear function* of x? What is its slope?

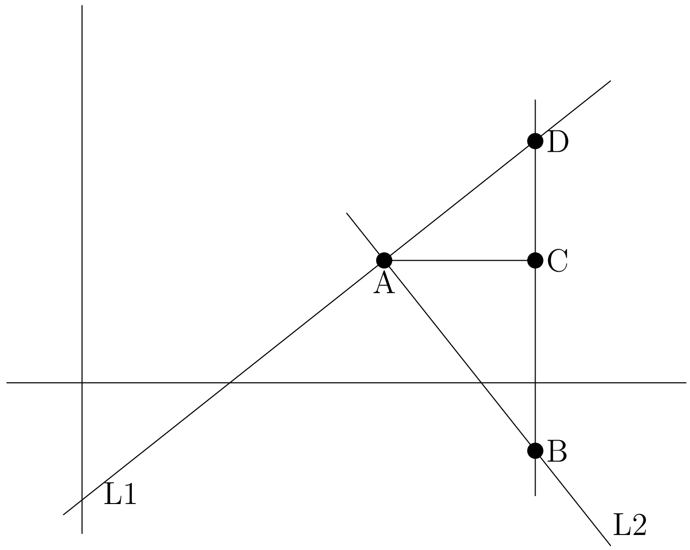
   (c) Find a formula for A in terms of x. Include units.

Parallel and Perpendicular Lines

5. Given two linear functions $f(x) = m_1 x + c_1$ and $g(x) = m_2 x + c_2$, how can you tell if the two lines are *parallel*?

6. Suppose that the line l is parallel to $y = \frac{1}{2}x - 2$ and passes though the point $(1, 2)$. Find its equation.

7. Use the picture below, the given information, what you have learned in high school geometry and the hints given below the picture to show that if two lines are perpendicular ($\perp$) then the product of their slopes is -1. In the picture below, the slope of L1 is m_1, the slope of L2 is m_2, CD $\perp$ AC, and the length of AC $= 1$.



   (a) L1 $\perp$ L2 $\Leftrightarrow$ $\angle$DAB is a right angle $\Leftrightarrow$ $(AB)^2 + (\quad)^2 = (\quad)^2$.

   (b) If the coordinates of A are (x_0, y_0) then the coordinates of C are (,), the coordinates of D are (x_0+1,) and the coordinates of B are (,).

   (c) By writing m_1 and m_2 in terms of CD, BC, and AC, and using Pythagoras explain why

   - $(AB)^2 = 1 + m_2^2$

   - $(AD)^2 = 1 + m_1^2$

   - $(BD)^2 = (m_1 - m_2)^2$

   (d) So in terms of x_0, y_0, m_1 and m_2, L1 $\perp$ L2 if and only if

   $$\underline{\hspace{6cm}} = \underline{\hspace{3cm}}.$$

   (e) Simplify the expression above to show that $m_1 \cdot m_2 = -1$.

8. Find an equation for the line l that is perpendicular to the line $y = \frac{1}{3}x + 2$ and passes through the point $(1, 2)$.

Proportionality

- We say that y is *directly proportional* to x if there is a non-zero constant k such that
$$y = kx;$$
- We say that y is *inversely proportional* to x if there is a non-zero constant k such that
$$y = \frac{k}{x}.$$

In either case, k is called the *constant of proportionality.*

9. The following law of gravitation was suggested by Ismael Bullialdus in 1645: "The gravitational attraction force (F) between two point masses is directly proportional to the product of their masses and inversely proportional to the square of their separation distance (d)." Assuming the masses are m_1 and m_2, write this as a formula.

Homework Problems

1. Given the line $s(t) = 3t - 4$, answer the following.
   (a) Find an equation for the line through the point $(1, 3)$ that is parallel to the graph of $s(t)$.
   (b) Find an equation for the line through $(1, 3)$ that is perpendicular to the graph of $s(t)$.
   (c) Let $g(x) = s(2x)$. First, give a formula for $g(x)$, then find an equation of the line though the point $(0, 0)$ that is parallel to the graph of $g(x)$.
2. Given that a triangle ABC in the coordinate plane has its vertices at the points $(1, 4)$, $(4, 8)$, and $(5, 1)$, show that triangle ABC is a right triangle.
3. Consider the piecewise defined function
$$h(t) = \begin{cases} t^2 - 1, & t \leq -2, \\ t + 5, & -2 < t < 1, \\ 3 - t^2, & t \geq 1. \end{cases}$$
   (a) Sketch a graph of $h(t)$.
   (b) Evaluate $h(-3)$, $h(-2)$, $h(0)$, $h(1)$, and $h(3)$.
   (c) Let $j(z) = 2h(z)$. Evaluate $j(-3)$, $j(-2)$, $j(0)$, $j(1)$, and $j(3)$.
   (d) Let $l(z) = h(z + 1)$. Evaluate $l(-3)$, $l(-2)$, $l(0)$, $l(1)$, and $l(3)$.

4. Explain why, for any given straight line, you can choose any two points to compute the slope, i.e. that no matter which two points you pick you always get the same slope. The picture below and what you learned in high school geometry should help you solve this problem.