Functions

A **function** is a rule that takes numbers from a set A and assigns them a number in a set B. The set A is called the *domain*. The *range*, of f is the set of points in B that are hit by f. The input is called the *independent variable* and the output is called the dependent variable. The *graph* of f is the set of points $(x, f(x))$ in the xy-plane.

Examples

- The following table defines a function with domain $\{15, 16, 17, 18, 19\}$ and range $\{88, 90, 93, 96, 98\}$:

Date in August	15	16	17	18	19
High Temp in Durham (°F)	88	90	93	96	98

- The function $f(x) = x^2$ has domain all real numbers, and range all non-negative numbers. Why isn't the range all real numbers as well?

- What are the domain and range of the following functions?
  - $g(x) = \frac{1}{x}$
  - $f(x) = \sqrt{x-3}$

- Explain why $s(x) = \pm\sqrt{x}$ is not a function.

The Meaning of Domains and Ranges

Much of the time, we deal not with abstract functions (like $f(x) = x^2$), but rather with functions whose domain and range have meaning. For example, in the Cancer Mortality lab we considered the function whose domain was Index of Exposure, and whose range was Cancer Mortality. Very often, the domain and range are assigned *units*. For example, the units of Cancer Mortality in the lab were cancer deaths per $20,000$ people per year.

Questions

1. Suppose $E(t)$ is is a function describing the number of students enrolled in our section of 105L as a function of time.
□Regrade

   (a) What would be sensible units for the domain of the function?

   (b) What would be a suitable domain?

   (c) If enrollment is capped at 45 students,what would be a suitable range? In what units?

2. Let $P = f(d)$ where P is the price of a particular stock (in dollars) d days from now. (f is the function relating P and d.) Describe in words what each of the following expressions mean. Also, indicate the units that any numbers or variables should have.
□Regrade

   (a) $f(3)$

   (b) $f(3) + 5$

   (c) $f(d + 3)$

   (d) Write an expression for an amount which is five dollars less than twice the stock's price ten days ago.

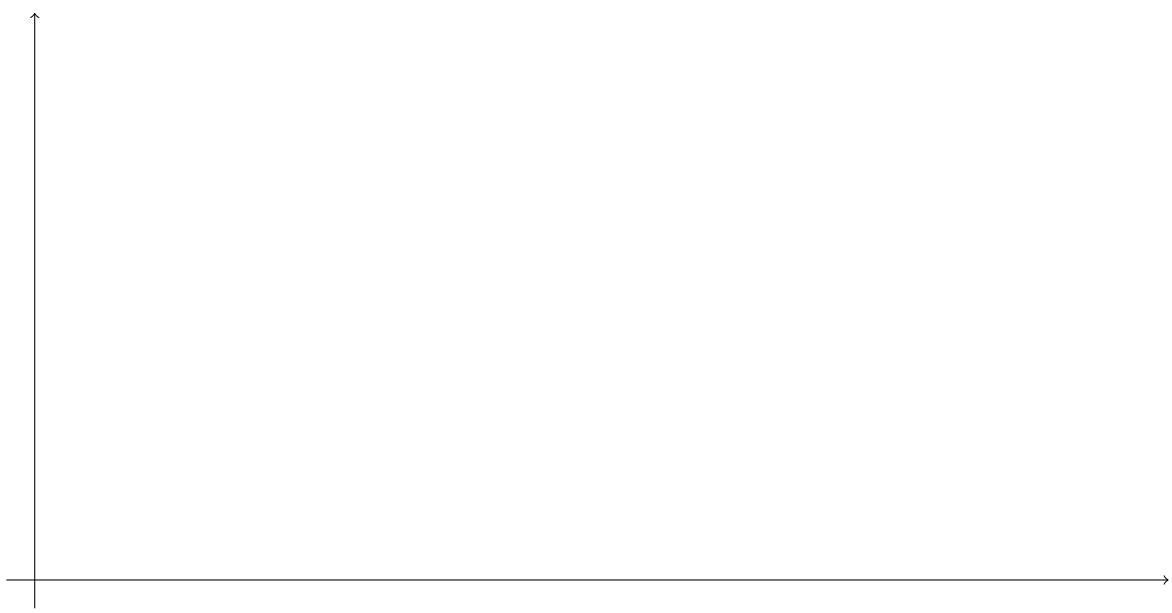
Graphs

Drawing the graphs of a function is often the best way to understand its behavior.

Questions

3. □Regrade Suppose you leave your dorm at 10am to walk to class. A little after you leave, you bump into a friend of yours and stand to chat for a while. Then you keep walking toward class. Suddenly, you realize you forgot your textbook in your dorm room, check the time and realize you're going to be late. You run back to the dorm as fast as you can, grab your book and run back to class, arriving at 10:20am.

Suppose $D(t)$ gives your distance from the dorm (in meters, say) at time t minutes after 10am. Draw a possible graph of $D(t)$ on the axes below. Label the axes with variables and units. No need to put a scale on the vertical axes. Take care to show the differences between walking and running.



4. □Regrade The following table gives 2016 Federal Income Tax in terms of Income (assuming a standard deductions, a personal exemption, and a taxpayer filing as a single person).

Taxable Income (I)	Income Tax (T)
up to \$10,350	\$0
between \$10,350 and \$19,625	10% of income over \$10,350
between \$19,625 and \$48,000	\$927.50 plus 15% of income over \$19,625
between \$48,000 and \$101,500	\$5,183.75 plus 25% of income over \$48,000
between \$101,500 and \$200,500	\$18,558.75 plus 28% of income over \$101,500
between \$200,500 and \$423,700	\$46,278.75 plus 33% of income over \$200,500
between \$423,700 and \$425,400	\$119,934.75 plus 35% of income over \$423,700
more than \$425,400	\$120,529.75 plus 39.6% of income over \$425,400

(a) What is your tax if your income is:

i. \$10,350?

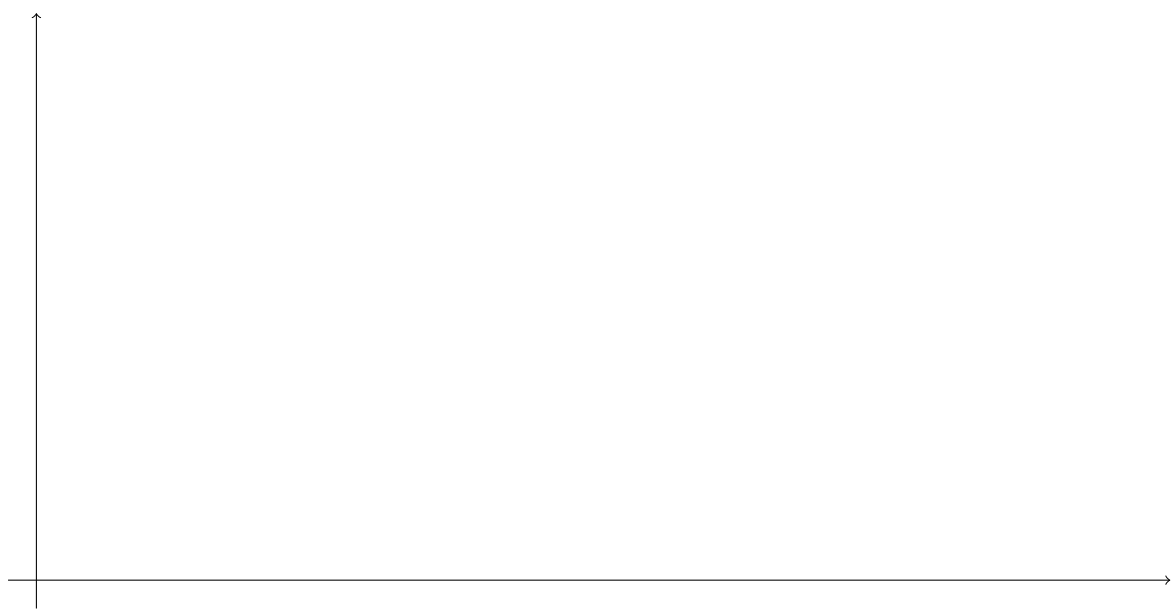
ii. \$12,000?

iii. \$47,999?

iv. \$48,001?

(Note: If you did this right, your last two answers should be very similar. If your income goes up \$2, your tax shouldn't go up much, even if you jump to a different tax bracket!)

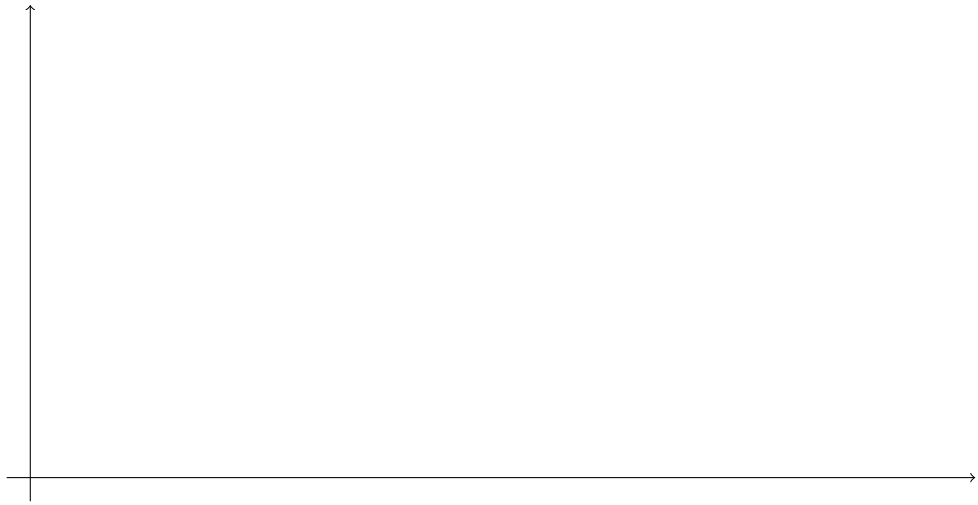
(b) Draw a graph of Taxable Income vs. Income tax on the axes below for incomes below \$100,000. Label the axes and put scales on them.



(c) The *marginal tax rate* at each income level is the percentage expressed in each of the rows of the table above. That is, at a given income level, it tells you how much more you'll pay in taxes if your income increases by a dollar. For example, if your taxable income is between \$48,000 and \$101,500, your marginal tax rate is 15%, meaning you will pay \$0.15 more in taxes for each dollar your income increases. What property of each graph segment in your graph above shows the marginal tax rate?

(d) Suppose that your income is x dollars, where $101,500 < x < 200,500$. Write a formula for your income tax.

5. For this problem consider the following story: You go for a jog to a nearby coffee shop
□Regrade where you sit for a few minutes having a cup of coffee. You then walk halfway home and run the rest of the way. Draw a graph of your distance from home as a function of time. Take care to show the differences between jogging, walking, and running. Label your axes, including suitable units.



6. For each of the following mathematical relationships between real numbers p and q
□Regrade answer the following questions: Is p a function of q? If so, what is the range? If not, why not?

(a) $p^2 + 2 = q$ (b) $p = q^2 + 2$ (c) $p + q = 2$ (d) $p^2 + q^2 = 1$

7. Circle all the tables in which w is a function of q.
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q	w
1	2
2	6.5
3	12
4	-7
5	2

q	w
2	5
4	9
6	4
8	8
4	12

q	w
π	1
e	2
1	3
5.6	4
-2	5

8. if the temperature in $^\circ F$ is F, then the temperature in $^\circ C$ is given by $C = \frac{5}{9}(F - 32)$.
□Regrade At what temperature is the degrees Celsius the same as the degrees Fahrenheit? Show all your work.