

Functions

A **function** is a rule that takes numbers from a set A and assigns them a number in a set B . The set A is called the *domain*. The *range*, of f is the set of points in B that are hit by f . The input is called the *independent variable* and the output is called the dependent variable. The *graph* of f is the set of points $(x, f(x))$ in the xy -plane.

Examples

The following table defines a function with domain $\{15, 16, 17, 18, 19\}$ and range $\{88, 90, 93, 96, 98\}$:

Date in August	15	16	17	18	19
High Temp in Durham ($^{\circ}\text{F}$)	88	90	93	96	98

1. The function $f(x) = x^2$ has domain all real numbers, and range all non-negative numbers. Why isn't the range all real numbers as well?

Questions

2. What are the domain and range of the following functions?

(a) $g(x) = \frac{1}{x}$

(b) $f(x) = \sqrt{x - 3}$

3. Explain why $s(x) = \pm\sqrt{x}$ is not a function.

The Meaning of Domains and Ranges

Much of the time, we deal not with abstract functions (like $f(x) = x^2$), but rather with functions whose domain and range have meaning. For example, in the Cancer Mortality lab we considered the function whose domain was Index of Exposure, and whose range was Cancer Mortality. Very often, the domain and range are assigned *units*. For example, the units of Cancer Mortality in the lab were cancer deaths per 20,000 people per year.

Questions

4. Suppose $E(t)$ is a function describing the number of students enrolled in our section of 105L as a function of time.
 - (a) What would be sensible units for the domain of the function?
 - (b) What would be a suitable domain? (assume we can't see into the future!)
 - (c) What would be a suitable range? In what units?

5. The volume of a sphere of radius r is given by $V(r) = \frac{4}{3}\pi r^3$. If the radius r is measured in centimeters, what are the units of the range of the function?

6. Let $P = f(d)$ where P is the price of a particular stock (in dollars) d days from now. (f is the function relating P and d .) Describe in words what each of the following expressions mean. Also, indicate the units that any numbers or variables should have.
 - (a) $f(3)$
 - (b) $f(3) + 5$
 - (c) $f(d + 3)$
 - (d) Write an expression for an amount which is five dollars less than twice the stock's price ten days ago.

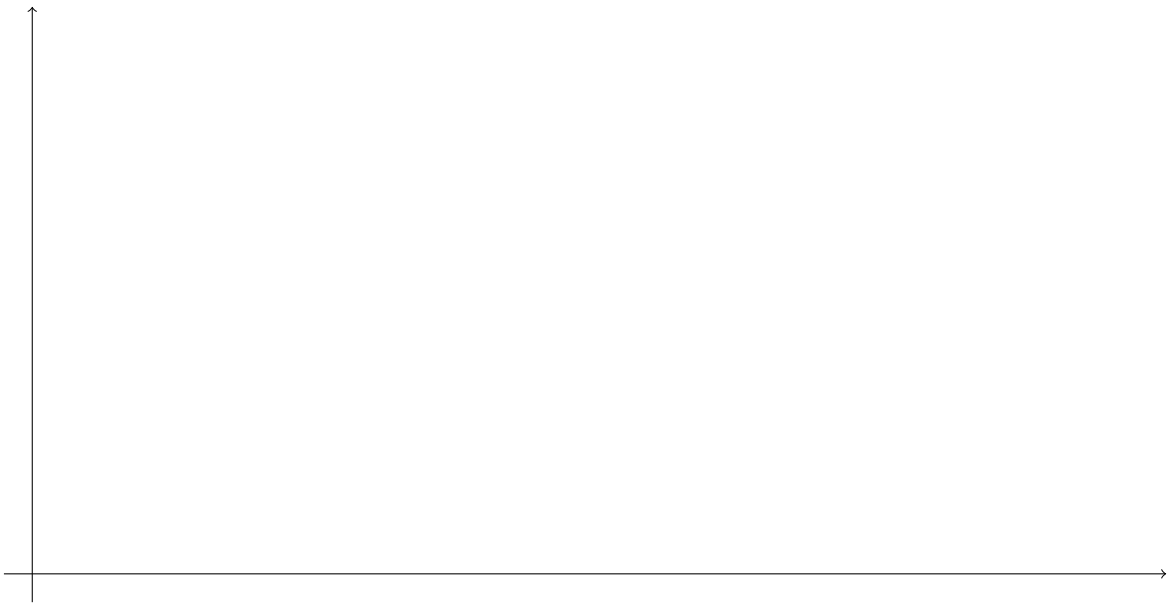
Graphs

Drawing the graphs of a function is often the best way to understand its behavior.

Questions

7. Suppose you leave your dorm at 10am to come to class. A little after you leave, you bump into a friend of yours and stand to chat for a while. Then you keep walking toward class. Suddenly, you realize you forgot your textbook in your dorm room, check the time and realize you're going to be late. You run back to the dorm as fast as you can, grab your book and run back to class, arriving at 10:20am.

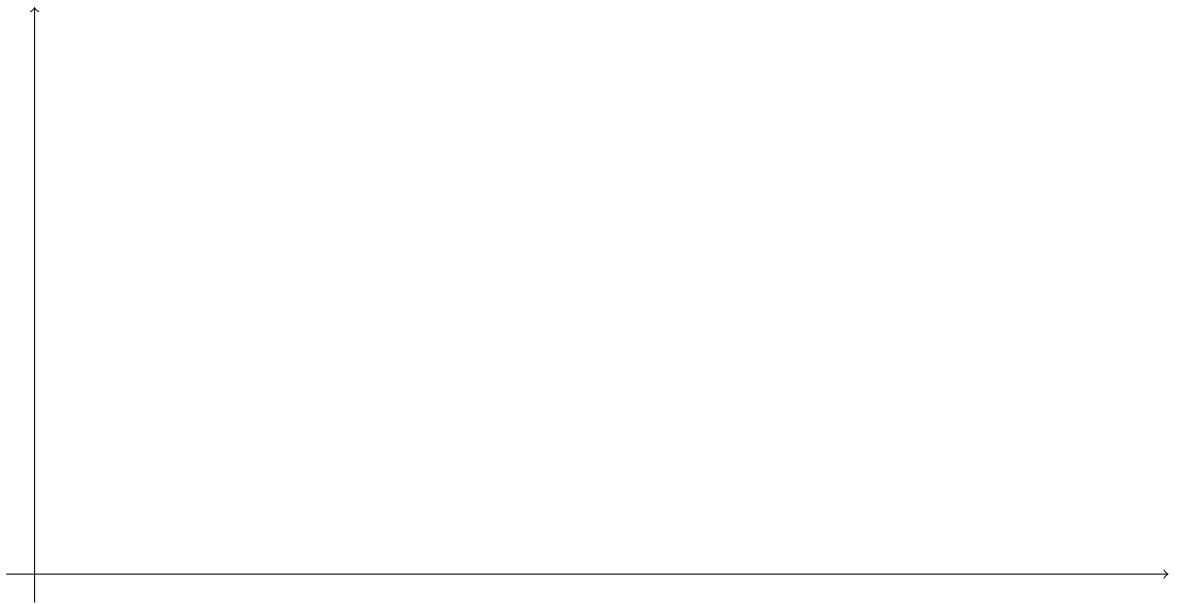
Suppose $D(t)$ gives your distance from the dorm (in meters, say) at time t minutes after 10am. Draw a possible graph of $D(t)$ on the axes below. Label the axes with variables and units. No need to put a scale on the vertical axes.



8. The following table gives 2016 Federal Income Tax in terms of Income (assuming a standard deductions, a personal exemption, and a taxpayer filing as a single person).

Taxable Income (I)	Income Tax (T)
up to \$10,350	\$0
between \$10,350 and \$19,625	10% of income over \$10,350
between \$19,625 and \$48,000	\$927.50 plus 15% of income over \$19,625
between \$48,000 and \$101,500	\$5,183.75 plus 25% of income over \$48,000
between \$101,500 and \$200,500	\$18,558.75 plus 28% of income over \$101,500
between \$200,500 and \$423,700	\$46,278.75 plus 33% of income over \$200,500
between \$423,700 and \$425,400	\$119,934.75 plus 35% of income over \$423,700
more than \$425,400	\$120,529.75 plus 39.6% of income over \$425,400

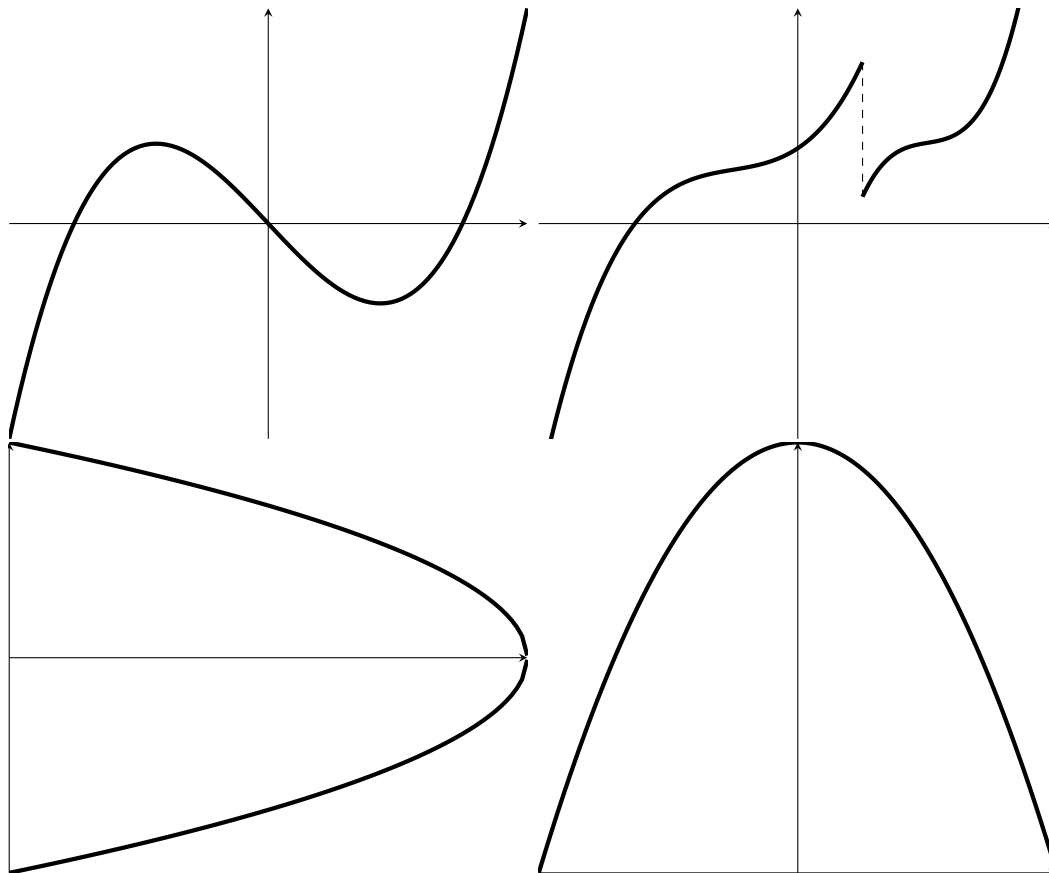
- (a) What is your tax if your income is:
- i. \$10,350?
 - ii. \$12,000?
 - iii. \$47,999?
 - iv. \$48,001?
- (b) Draw a graph of Taxable Income vs. Income tax on the axes below for incomes below \$100,000. Label the axes and put scales on them.



- (c) Write down a piecewise function giving income tax as a function of income below \$100,000. Be sure to define all your variables!
- (d) (Bonus questions, for general knowledge)
- i. This income tax system is often described as a ‘progressive’ tax (as opposed to a ‘regressive’ tax). Find out what that means.
 - ii. Sales tax is often a fixed-percentage tax on goods you buy. Some people consider sales tax to be a tax to be neither progressive nor regressive. Explain this. Why is sales tax considered by some to be a regressive tax?

Homework Problems

1. Which of the following are graphs of functions?



2. For this problem consider the following story: You go for a jog to a nearby coffee shop where you sit for a few minutes having a cup of coffee. You then walk halfway home and run the rest of the way. Draw two sets of axes, then sketch graphs representing your distance from home as a function of time and your velocity as a function of time. Be sure to label the axes of your graphs and give appropriate units.
3. For each of the following mathematical relationships between real numbers p and q answer the following questions: Is p a function of q ? If so, what is the range? If not, why not?

(a) $p^2 + 2 = q$ (b) $p = q^2 + 2$ (c) $p + q = 2$ (d) $p^2 + q^2 = 1$

4. In which of the following tables is w a function of q ?

q	w
1	2
2	6.5
3	12
4	-7
5	2

q	w
2	5
4	9
6	4
8	8
4	12

q	w
π	1
e	2
1	3
5.6	4
-2	5

5. Using the relationship in the table below, is Income Tax a function of Taxable Income? Is Taxable Income a function of Income Tax?

Taxable Income	Income Tax
up to \$10,000	\$0
between \$10,000 and \$50,000	15% of income over \$10,000
\$50,000 or more	\$6,000 plus 27% of income over \$50,000

6. At what temperature is the degrees Celsius the same as the degrees Fahrenheit?
7. What is meant when we say that two varying quantities, say velocity and time, are proportional to each other? In the following table, is velocity proportional to time?

Velocity	7	11	15	19	23
Time	3	5	7	9	11

8. From the 2007 NC Tax tables:

If your filing status is single, and taxable income is

more than	but not over	The tax is
\$0	\$12,750	6% of the amount on line 13, D-400
\$12,750	\$60,000	\$765 + 7% of the amount over \$12,750
\$60,000	\$120,000	\$4,072.50 + 7.75% of the amount over \$60,000
\$120,000		\$8572.50 + 8% of the amount over \$120,000

Is this a function?