Past Differentiation...

The differentiation rules we’ve learned so far:

1. Power Rule:

2. Sum Rule:

3. Difference Rule:

4. Product Rule:

5. Quotient Rule:

Motivation!

Questions

1. With the tools we have, how do we find \( \frac{d}{dx}(x^3 + 1)^3 \)? Is there any easier way?

2. (a) (From the coursepack): Let \( L(E) \) be the life expectancy (years) of a person with \( E \) years of education. Let \( E(i) \) be the average number of years of education obtained by people whose parents have an average annual income of \( i \) dollars. Describe the function \( L(E(i)) \) in practical terms. Include units in your answer.

   (b) What are the units of \( \frac{dL}{dE}, \frac{dE}{di}, \) and \( \frac{dL}{di} \)?
The Chain Rule

More general question: How do we differentiate the composition of two functions (functions inside of functions)?

The Chain Rule
If \( f(x) \) and \( g(x) \) are differentiable, then

\[
\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)
\]

If we let \( z = g(x) \) and \( y = f(z) \), then \( y = f(g(x)) \) and we can write the chain rule in the following way:

\[
\frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx}
\]

Questions  Differentiate the functions below:

1. \((x^3 + 1)^3\) (How does this compare to our answer above?)

2. \((x^2 + 1)^{100}\)

3. Find \(\frac{d}{dx}(8x^2 + 2)^7\).

4. Find \(\frac{d}{dx}\left(\frac{1}{x^2} + x + 1\right)^3\).
5. Find \( \frac{d}{dx} \left( \frac{(2x+3)^2}{(x+1)^3} \right)^{\frac{3}{2}} \).

**Question**  A new proof of the quotient rule using the chain and product rules:

**Question**  Find the equation of the tangent line to \( f(x) = \left( \frac{2}{x} - 1 \right)^3 \) at the point where \( x = 3 \).

**Question**  The Triple Chain Rule: What about the composition of three functions, \( f(g(h(x))) \)? How would we differentiate that?