Transforming Functions by Addition

1. Suppose $f(x) = x^2$. Then write out expressions for:
   
   (a) $f(x) + 2$

   (b) $f(x + 2)$

2. On the axes below, draw the graphs of $f(x)$, $y = f(x) + 2$ and $y = f(x + 2)$. Label your graphs and axes.

3. Fill in the blanks:
   
   (a) If we start from the graph of a function $f(x)$, the graph of the function $f(x) + a$ has exactly the same shape, but is shifted __________ by ______ units.

   (b) If we start from the graph of a function $f(x)$ the graph of the function $f(x + a)$ has exactly the same shape, but is shifted __________ by ______ units.

4. Given the graph of a function $f(x)$ below, draw the functions $f(x + 3)$ and $f(x) - 8$ on the same axes.
Transforming Functions by Multiplication

1. Suppose \( f(x) = x(x - 1)(x + 1) \). Then write out expressions for:

   (a) \( f(2x) \)
   (b) \( 2f(x) \)

2. On the axes below, draw the graphs of \( f(x) \), \( f(2x) \) and \( 2f(x) \). Label your axes and graphs.

3. Fill in the blanks:

   (a) If we start from the graph of a function \( f(x) \), the graph of the function \( af(x) \) has the same shape, but is stretched ________ by a factor of ______.

   (b) If we start from the graph of a function \( f(x) \) the graph of the function \( f(ax) \) has exactly the same shape, but is stretched ________ by a factor of ______.

4. Given the graph of a function \( f(x) \) below, draw the functions \( f(2x) \) and \( 2f(x) \) on the same axes.
Function Reflections

1. Suppose \( f(x) = x(x - 1) \). Then write out expressions for:
   
   (a) \( f(-x) \)
   
   (b) \(-f(x)\)

2. On the axes below, draw the graphs of \( f(x) \), \(-f(x)\) and \( f(-x)\). Label your axes and graphs.

3. Fill in the blanks:
   
   (a) If we start from the graph of a function \( f(x) \), the graph of the function \(-f(x)\) is the same graph, but \[\underline{\quad}\] in the \[\underline{\quad}\]-axis.

   (b) If we start from the graph of a function \( f(x) \), the graph of the function \( f(-x) \) is the same graph, but \[\underline{\quad}\] in the \[\underline{\quad}\]-axis.

4. Given the graph of the function \( f(x) \) below, draw the functions \(-f(x)\) and \( f(-x)\) on the same axes.

-4  -2  2  4

-20  20
Putting it all Together

**Question**  Consider the graph of $f(x)$ in the previous question.

1. Can you figure out a possible formula for it? (Hint: think back to polynomials.)

2. Write down a formula for $f(2(x - 1))$. Simplify it, but do not FOIL. What are the roots of $f(2(x - 1))$?

3. On the axes below, draw the graphs of $f(x)$ and $f(2(x - 1))$.

4. By considering the zeros of $f(x)$ in part (1), of $f(2(x - 1))$ in part (2), and the graphs in part (3), decide which of the following statements is true and which is false:

   (a) To get from $f(x)$ to $f(2(x - 1))$ you first compress along the $x$-axis by a factor of 2, then shift to the right by 1.

   (b) To get from $f(x)$ to $f(2(x - 1))$ you first shift to the right by 1, then compress along the $x$-axis by a factor of 2.

**Question**  Given a graph of $f(x)$, describe how you would go about drawing the graph of $f(6x - 3)$.
Question  Given the graph of $f(x)$ below, draw the graph of $-2f(0.5x) - 3$. (Hint: you might want to do this step-by-step using the axes below. Be sure to label each with what you’re drawing!) As a bonus: can you identify (with a formula) the function you get at the end? Can you use that to identify the function of the original graph?