Review

Question Assuming $x$ and $y$ are functions of $t$, compute the following derivatives. Note: these are implicit derivatives!

1. $\frac{d}{dt}\sqrt{x^2 + y^2}$

2. $\frac{d}{dt}x^2y$

Related Rates

Questions

1. You are blowing up a spherical balloon at a rate of $50\text{cm}^3/\text{min}$.
   (a) What is the volume $V$ of the balloon when its radius is $r$ cm?

   (b) Since the radius is changing, it is a function of time. Differentiate your answer with respect to time, $t$.

   (c) Use the fact that the rate of change of volume is $50\text{cm}^3/\text{min}$ and the previous answer to find the rate of change of the radius when the radius is 5cm.

   (d) What about when the radius is 10cm?
(e) Is the radius changing faster when the balloon has radius 5cm or radius 10cm? Use the picture to explain why this makes sense. Note that the shaded areas are the same.

2. Suppose two cars leave an intersection simultaneously. One travels north at 30 mi/hr, while the other travels east at 40 mi/hr. How fast is the distance between the two changing after one minute?
3. The ideal gas law says \( PV = k \). If the quantity of gas and temperature remain fixed, this becomes Boyle’s Law. Then, the pressure \( P \) and the volume \( V \) satisfy \( PV = k \) for some constant \( k \). Suppose a certain quantity of gas occupies 10 cubic centimeters at a pressure of 2 atmospheres. If the pressure is increasing at a rate of 0.05 atmospheres per minute when the pressure is 2 atmospheres, find the rate at which the volume is changing at that moment. What are the units of your answer?

4. Suppose that we steal the world’s largest marshmallow\(^1\). It is a cylinder of volume 7300 cubic feet with a base radius of 12.5 feet. In order to make the world’s largest s’more out of it, we heat it and find that the radius is changing at a rate of 0.6 feet per minute and the height is changing at a rate at a rate of 0.4 feet per minute. How fast is the volume changing just as we start to heat the marshmallow?

\(^1\)Found in Noble County, Indiana and weighing 2300 pounds