Definitions

- Experiment:

- Sample Space:

- Event:

- Probability:

Examples For each of the following examples, write down the sample space:

1. Flipping a (fair) coin once.

2. Flipping a (fair) coin twice.

3. Rolling a (fair) 6-sided die once.

Principles These are the two most fundamental principles of probability:

1.

2.

Notation

- \( P(A \cap B) = \)

- \( P(A \cup B) = \)

Question What is the difference between an event and an outcome?
Example: Rolling Two Dice

Consider an experiment where we roll two dice.

The sample space is the set of any combination of rolls as listed below, and each individual outcome has a probability of _____.

1, 1 1, 2 1, 3 1, 4 1, 5 1, 6
2, 1 2, 2 2, 3 2, 4 2, 5 2, 6
3, 1 3, 2 3, 3 3, 4 3, 5 3, 6
4, 1 4, 2 4, 3 4, 4 4, 5 4, 6
5, 1 5, 2 5, 3 5, 4 5, 5 5, 6
6, 1 6, 2 6, 3 6, 4 6, 5 6, 6

1. Let $A$ denote the event of rolling a 3 on the first die. Let $B$ denote the event that the sum of the two rolls is exactly 7.

(a) Write down the subset of the sample space that corresponds to event $A$:

(b) Write down the subset of the sample space that corresponds to event $B$:

(c) $P(A) =$   $P(B) =$

$P(A \cap B) =$   $P(A \cup B) =$

2. Let $C$ denote the event of rolling a 3 on the first die. Let $D$ denote the event of rolling an even number on the second die.

$P(C) =$   $P(D) =$

$P(C \cap D) =$   $P(C \cup D) =$

3. Let $E$ denote the event of rolling a 2 on the first die. Let $F$ denote the event that the sum of the two rolls is at least 8.

$P(E) =$   $P(F) =$

$P(E \cap F) =$   $P(E \cup F) =$
Independence and the Addition Rule

**Independence:** We say the events $A$ and $B$ are independent if and only if

$$P(A \cap B) = \quad .$$

**Example** Are events $A$ and $B$ from the last example independent? What about events $C$ and $D$? $E$ and $F$?

**Addition Rule:** Use the picture below to write a formula that gives us $P(A \cup B)$ as some combination of $P(A)$, $P(B)$, and $P(A \cap B)$.

The addition rule is

$$P(A \cup B) = \quad .$$

Check that the addition rule works on the events in the last example:

- $P(A \cup B) =$
  
  Right-hand side of addition rule =

- $P(C \cup D) =$
  
  Right-hand side of addition rule =

- $P(E \cup F) =$
  
  Right-hand side of addition rule =
Example: Roulette Table

A roulette wheel has two green spaces, 18 red spaces, and 18 black spaces. If the wheel is spun three times, then find the probability that

1. all three spins come up red

2. none of the spins come up red

3. at least one spin was red

Question: If the last ten spins were red, what is the probability of the next spin being black?