Exam 1 - Practice

These instructions will be on the test:

- Do not open this test booklet until you are directed to do so.
- You will have 50 minutes to complete the exam. If you finish early go back and check your work.
- This exam is closed book.
- You may use a calculator for arithmetic purpose only. No calculator matrix computations are allowed.
- Throughout the exam, show your work so that your reasoning is clear. Otherwise no credit will be given. Circle your answers.
- Do not spend too much time on any one problem. Read them all through first and attack them in an order that allows you to make the most progress.

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Name: __________________________
1. (25 points) Consider the matrix

\[
A = \begin{pmatrix}
1 & 2 & 5 & 0 \\
1 & 2 & 4 & 2 \\
0 & -1 & 0 & 8 \\
-1 & -3 & -1 & -1
\end{pmatrix}.
\]

The goal of this question is to solve the equation \( A\vec{x} = [12, 1, -30, 6]^T \).

(a) Carry out Gaussian reduction with maximal partial pivoting to show that the upper triangular component of the \( PA = LU \) decomposition is

\[
U = \begin{pmatrix}
1 & 2 & 5 & 0 \\
0 & -1 & 0 & 8 \\
0 & 0 & 4 & -9 \\
0 & 0 & 0 & \frac{1}{4}
\end{pmatrix},
\]

and find the \( L \) and \( P \) components. You may want to augment \( A \) with the vector above to make the next part of this question easier.
(b) Suppose that $A\vec{x} = [12, 1, -30, 6]^T$. Find $\vec{x}$.

2. (5 points) Consider the matrix $B = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 7 & 1 \\ 2 & 4 & 1 \end{pmatrix}$. Is $B$ invertible? If so, find its inverse. If not, explain why.
3. (15 points) Consider the matrix
\[
D = \begin{pmatrix}
1 & 2 & 3 & 2 & 14 & 9 \\
0 & 0 & 0 & 2 & 10 & 6 \\
0 & 0 & 0 & 0 & 0 & 3
\end{pmatrix}.
\]

Note that this matrix is in row echelon form.

(a) Fill in the blanks:
   i. The column space \( C(D) \) is a _____ dimensional subspace of \( \mathbb{R}^m \), where \( m = ____ \).
   ii. The nullspace \( N(D) \) is a _____ dimensional subspace of \( \mathbb{R}^n \), where \( n = ____ \).

(b) Write down a basis for \( C(D) \).

(c) Compute the row-reduced echelon form of \( D \).

(d) Find a basis for \( N(D) \).

(e) Let \( \vec{x}_p \) be the particular solution of \( D\vec{x} = \vec{c} \) for some \( \vec{c} \in C(D) \). Write down an expression giving all possible solutions of this equation.
4. (5 points)

(a) Briefly explain why for any two matrices $E$ and $F$, the column space of $EF$ is contained in the column space of $E$.

(b) Find a matrix $F$ with no zero entries such that the column space of $EF$ is not equal to the column space of $E$. You do not need to compute the column spaces of either matrix in order to answer this question, but you can use the following matrix if you wish:

$$E = \begin{pmatrix} 1 & 2 & 3 \\ -1 & -2 & 4 \\ 0 & 0 & 7 \end{pmatrix}.$$