Duke University

Math 218: Matrices and Vector Spaces

## Exam 1 - Practice

These instructions will be on the test:

- Do not open this test booklet until you are directed to do so.
- You will have 50 minutes to complete the exam. If you finish early go back and check your work.
- This exam is closed book.
- You may use a calculator for arithmetic purpose only. No calculator matrix computations are allowed.
- Throughout the exam, show your work so that your reasoning is clear. Otherwise no credit will be given. Circle your answers.
- Do not spend too much time on any one problem. Read them all through first and attack them in an order that allows you to make the most progress.

| Problem | Points | Grade |
| :---: | :---: | :---: |
| 1 | 25 |  |
| 2 | 5 |  |
| 3 | 9 |  |
| 4 | 6 |  |
| 4 | 5 |  |
| Total | 50 |  |

Name: $\qquad$

1. (25 points) Consider the matrix

$$
A=\left(\begin{array}{rrrr}
1 & 2 & 5 & 0 \\
1 & 2 & 4 & 2 \\
0 & -1 & 0 & 8 \\
-1 & -3 & -1 & -1
\end{array}\right)
$$

The goal of this question is to solve the equation $A \vec{x}=[12,1,-30,6]^{T}$.
(a) Carry out Gaussian reduction with maximal partial pivoting to show that the upper triangular component of the $P A=L U$ decomposition is

$$
U=\left(\begin{array}{rrrr}
1 & 2 & 5 & 0 \\
0 & -1 & 0 & 8 \\
0 & 0 & 4 & -9 \\
0 & 0 & 0 & -\frac{1}{4}
\end{array}\right)
$$

and find the $L$ and $P$ components. You may want to augment $A$ with the vector above to make the next part of this question easier.

NOTE: Maximal partial pivoting will not be tested on the actual test explicitly, but you should understand it!
(b) Suppose that $A \vec{x}=[12,1,-30,6]^{T}$. Find $\vec{x}$.
2. (5 points) Consider the matrix $B=\left(\begin{array}{rrr}1 & -1 & 0 \\ -1 & 7 & 1 \\ 2 & 4 & 1\end{array}\right)$. Is $B$ invertible? If so, find its inverse. If not, explain why.
3. ( 9 points) Which of the following is a vector subspace of $\mathbb{R}^{3}$ ? Justify your answers. Answers with no justification will receive no credit.
(a) All vectors $\left[b_{1}, b_{2}, b_{3}\right]^{T}$ such that $b_{1}+b_{2}-2 b_{3}=0$.
(b) All vectors $\left[b_{1}, b_{2}, b_{3}\right]^{T}$ such that $b_{1} \leq b_{2} \leq b_{3}$.
(c) All linear combinations of $[1,2,4]^{T}$ and $[2,3,4]^{T}$.
4. (6 points)
(a) Suppose that $\vec{v}=[1,1]^{T}$ and $\vec{w}=[1,5]^{T}$. Find a number $c$ so that $\vec{w}-c \vec{v}$ is perpendicular to $\vec{v}$.
(b) Suppose that $\vec{v}$ and $\vec{w}$ are both non-zero vectors. Find a formula for a number $c$ so that $\vec{w}-c \vec{v}$ is perpendicular to $\vec{v}$.
5. (5 points)
(a) Briefly explain why for any two matrices $E$ and $F$, the column space of $E F$ is contained in the column space of $E$.
(b) Find a matrix $F$ with no zero entries such that the column space of $E F$ is not equal to the column space of $E$. You do not need to compute the column spaces of either matrix in order to answer this question, but you can use the following matrix if you wish:

$$
E=\left(\begin{array}{rrr}
1 & 2 & 3 \\
-1 & -2 & 4 \\
0 & 0 & 7
\end{array}\right)
$$

