Exercises from Strang P. 190 4, 18, 25; P. 203 9, 14, 21, 28, 30; P. 215 2, 5-7, 11, 12, 17[1], 19, 20, 27, 31; P. 229 1, 9[2], 25, 28.

Extra Problem

You may use SAGE for any or all parts of this question. Be sure to show all your work. For example, if you used SAGE to compute the RREF of a matrix, show the RREF and explain what conclusions you draw from it that answer the relevant question.

Consider the equation $A\vec{x} = \vec{c}$ with

$$A = \begin{pmatrix} 1 & -1 & 5 & -4 \\ 2 & 0 & 4 & -2 \\ 3 & 2 & 0 & 3 \end{pmatrix} \text{ and } \vec{c} = \begin{pmatrix} 0 \\ 2 \\ 6 \end{pmatrix}$$

1. Show that $\vec{c} \notin C(A)$.
2. Compute $A^T A$ and $A^T \vec{c}$.
3. Why does the normal equation $A^T A\hat{x} = A^T \vec{c}$ not have a unique solution?
4. Find the general solution of the normal equation $\hat{x} = \hat{x}_p + \hat{x}_n$.
5. Find the projection $\vec{p}$ of $\vec{c}$ onto $C(A)$ and the error vector $\vec{e} \in N(A^T)$.
6. Find the projection matrix P onto $C(A)$, and check that $P\vec{c} = \vec{p}$. (Hint: see question 28 on Page 232 of the book.)
7. (Super challenge question - extra credit) Find the projection $\hat{x}_{\text{row}}$ of $\hat{x}_p$ onto the row space of A, and rewrite the general solution $\hat{x} = \hat{x}_{\text{row}} + \hat{x}_n$. This is the optimal way to write the solution to this equation, in the sense that you can separate $\hat{x}$ into its orthogonal components in $C(A^T)$ and $N(A)$, as well as $\vec{v}$ into its orthogonal components in $C(A)$ and $N(A^T)$.

[1] See the top of page 210. For once, this 'important' question is actually important!

[2] The points referred to are $(0, 0)$, $(1, 8)$, $(3, 8)$, and $(4, 20)$. Use Sage Cell to find the solution. Ignore the last part of the question. Instead, plot the points and the parabola, as well as the errors.