Exercises from Strang  P. 161 24, 25, 34; P. 175 7, 8, 12, 21, 22, 45\textsuperscript{[1]} P. 190 3, 4, 11, 18, 19, 24, 25; P 202 9, 10(a), 14, 17, 21.

Extra Problem

Use the Sage Cell Server to find the RREF of the following 7 \times 10 matrix. Also find the RREF of its transpose.

\[
\begin{pmatrix}
2 & -2 & 1 & 2 & -4 & -4 & 1 & -8 & 9 & 1 \\
-4 & -3 & 0 & 0 & 11 & -5 & -9 & 5 & -5 & 3 \\
-5 & -5 & 1 & 3 & -1 & -8 & -3 & -10 & 8 & 6 \\
3 & 3 & -1 & -2 & 2 & 5 & 0 & 9 & -9 & -4 \\
4 & 4 & 0 & 1 & -11 & 4 & 8 & -4 & 7 & -10 \\
2 & 2 & 0 & -3 & 5 & 4 & 1 & 6 & -3 & 1 \\
3 & 3 & 0 & -3 & 3 & 6 & 2 & 8 & -5 & -1
\end{pmatrix}
\]

You can copy and paste the following code to initialize the matrix:

\[
A=\text{matrix}([[-2,-2,1,2,-4,-4,1,-8,9,1],[-4,-3,0,0,11,-5,-9,5,-5,3],[-5,-5,1,3,-1,-8,-3,-10,8,6],[3,3,-1,-2,2,5,0,9,-9,-4],[4,4,0,1,-11,4,8,-4,7,-10],[2,2,0,-3,5,4,1,6,-3,1],[3,3,0,-3,3,6,2,8,-5,-1]])
\]

To find the RREF and transpose of a matrix A:

\[
A.\text{rref()}
\]
\[
A.T
\]

Then answer the following questions:

1. What is \(\text{rank}(A)\)?

2. Find bases for each of the four fundamental spaces of A.

3. The following code creates a 7 \times 11 matrix of zeros, then inserts A into its first 10 columns and the vector \(\vec{c} = \begin{pmatrix} -6, -12, -18, 13, 22, -1, 6 \end{pmatrix}^T\) into its last column, creating the augmented matrix \([A|c]\):

\[
B = \text{matrix}(7,11)
B[:,0:10]=A
B[:,10]=\text{vector}([-6, -12, -18, 13, 22, -1, 6])
\]

Use the RREF of the augmented matrix to find the particular solution \(x_p\) of \(A\vec{x} = \vec{c}\).

4. Since \(x_p \in \mathbb{R}^{10}\), it can be written as linear combination of basis vectors of which two spaces?

5. Explain how you would go about expressing \(x_p\) as a linear combination of those basis vectors.
   (For bonus points, or for some masochistic fun: carry this out...please don’t do it by hand...the numbers are very nasty...)

\textsuperscript{[1]}You may not use the dimension formula from Question 43, as we haven’t proved it. Instead, write down a basis for \(V\) and a basis for \(W\), then figure out why combining the two bases gives a linearly dependent set. Lastly, use the formal definition of linear dependence and a bit of algebra to find a non-zero vector that must be in both spaces. You may also want to do P 202 Problem 14 first.