Exercises from Strang  P. 142 Q 20, 21; 31, 33; P 159 4, 5, 10, 16, 22; 24, 25, 33, 34; P. 175 7, 8, 12, 16, 21, 22, 45

Extra Problem

Use the Sage Cell Server to find the RREF of the following 7 × 10 matrix. Also find the RREF of its transpose.

\[
\begin{bmatrix}
-2 & -2 & 1 & 2 & -4 & -4 & 1 & -8 & 9 & 1 \\
-4 & -3 & 0 & 0 & 11 & -5 & -9 & 5 & -5 & 3 \\
-5 & -5 & 1 & 3 & -1 & -8 & -3 & -10 & 8 & 6 \\
3 & 3 & -1 & -2 & 2 & 5 & 0 & 9 & -9 & -4 \\
4 & 4 & 0 & 1 & -11 & 4 & 8 & -4 & 7 & -10 \\
2 & 2 & 0 & -3 & 5 & 4 & 1 & 6 & -3 & 1 \\
3 & 3 & 0 & -3 & 3 & 6 & 2 & 8 & -5 & -1
\end{bmatrix}
\]

You can copy and paste the following code to initialize the matrix:

```python
A=matrix([[[-2,-2,1,2,-4,-4,1,-8,9,1],[-4,-3,0,0,11,-5,-9,5,-5,3],[-5,-5,1,3,-1,-8,-3,-10,8,6],[3,3,-1,-2,2,5,0,9,-9,-4],[4,4,0,1,-11,4,8,-4,7,-10],[2,2,0,-3,5,4,1,6,-3,1],[3,3,0,-3,3,6,2,8,-5,-1]]
```

To find the RREF and transpose of a matrix A:

```python
A.rref()
A.T
```

Then answer the following questions:

1. What is rank(A)?

2. Find bases for each of the four fundamental spaces of A.

3. The following code creates a 7 × 11 matrix of zeros, then inserts A into its first 10 columns and the vector \( \vec{c} = [-6, -12, -18, 13, 22, -1, 6] \)^T into its last column, creating the augmented matrix \([A|\vec{c}]\):

   ```python
   B = matrix(7,11)
   B[:,:10]=A
   B[:,10]=vector([-6, -12, -18, 13, 22, -1, 6])
   ```

   Use the RREF of the augmented matrix to find the particular solution \( x_p \) of \( Ax = \vec{c} \).

4. Since \( x_p \in \mathbb{R}^{10} \), it can be written as linear combination of basis vectors of which two spaces?

5. Explain how you would go about expressing \( x_p \) as a linear combination of those basis vectors.
   (For bonus points, or for some masochistic fun: carry this out...please don’t do it by hand...the numbers are very nasty...)

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1 Hint: see Question 4, same page
2 Note: B here is a vector, not a matrix!
3 You may not use the dimension formula from Question 43, as we haven’t proved it. Instead, write down a basis for \( V \) and a basis for \( W \), then figure out why combining the two bases gives a linearly dependent set. Lastly, use the formal definition of linear dependence and a bit of algebra to find a non-zero vector that must be in both spaces. You may also want to do P 202 Problem 14 first.