Exercises from Strang P. 142 Q 20, 21[1], 31, 33; P 159 4, 5, 10, 16, 22[2], 24, 25, 33, 34; P. 175 7, 8, 12, 16, 21, 22, 45[3];

Extra Problem

Use the Sage Cell Server to find the RREF of the following 7×10 matrix. Also find the RREF of its transpose.

$$\begin{pmatrix} -2 & -2 & 1 & 2 & -4 & -4 & 1 & -8 & 9 & 1 \\ -4 & -3 & 0 & 0 & 11 & -5 & -9 & 5 & -5 & 3 \\ -5 & -5 & 1 & 3 & -1 & -8 & -3 & -10 & 8 & 6 \\ 3 & 3 & -1 & -2 & 2 & 5 & 0 & 9 & -9 & -4 \\ 4 & 4 & 0 & 1 & -11 & 4 & 8 & -4 & 7 & -10 \\ 2 & 2 & 0 & -3 & 5 & 4 & 1 & 6 & -3 & 1 \\ 3 & 3 & 0 & -3 & 3 & 6 & 2 & 8 & -5 & -1 \end{pmatrix}$$

You can copy and paste the following code to initialize the matrix:

```
A=matrix([[-2,-2,1,2,-4,-4,1,-8,9,1],[-4,-3,0,0,11,-5,-9,5,-5,3],
[-5,-5,1,3,-1,-8,-3,-10,8,6],[3,3,-1,-2,2,5,0,9,-9,-4],[4,4,0,1,-11,4,8,-4,7,-10],
[2,2,0,-3,5,4,1,6,-3,1],[3,3,0,-3,3,6,2,8,-5,-1]])
```

To find the RREF and transpose of a matrix A:

```
A.rref()
A.T
```

Then answer the following questions:

1. What is $rank(A)$?
2. Find bases for each of the four fundamental spaces of A.
3. The following code creates a 7×11 matrix of zeros, then inserts A into its first 10 columns and the vector $\vec{c} = [(-6, -12, -18, 13, 22, -1, 6)]^T$ into its last column, creating the augmented matrix $[A|c]$:

```
B = matrix(7,11)
B[:,:10]=A
B[:,10]=vector([-6, -12, -18, 13, 22, -1, 6])
```

   Use the RREF of the augmented matrix to find the particular solution x_p of $A\vec{x} = \vec{c}$.

4. Since $x_p \in \mathbb{R}^{10}$, it can be written as linear combination of basis vectors of which two spaces?
5. Explain how you would go about expressing x_p as a linear combination of those basis vectors. (For bonus points, or for some masochistic fun: carry this out...please don't do it by hand...the numbers are very nasty...)

[1] Hint: see Question 4, same page

[2] Note: B here is a vector, not a matrix!

[3] You may **not** use the dimension formula from Question 43, as we haven't proved it. Instead, write down a basis for V and a basis for W, then figure out why combining the two bases gives a linearly dependent set. Lastly, use the formal definition of linear dependence and a bit of algebra to find a non-zero vector that must be in both spaces. You may also want to do P 202 Problem 14 first.