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9. (a) $\left(\begin{array}{ccc}1 & 2 & 4 \\ -2 & 3 & 1 \\ -4 & 1 & 2\end{array}\right)\left(\begin{array}{l}2 \\ 2 \\ 3\end{array}\right)=\left(\begin{array}{c}1 \cdot 2+2 \cdot 2+4 \cdot 3 \\ -2 \cdot 2+3 \cdot 2+1 \cdot 3 \\ -4 \cdot 2+1 \cdot 2+2 \cdot 3\end{array}\right)=\left(\begin{array}{c}16 \\ 5 \\ 0\end{array}\right)$
(b) $\left(\begin{array}{llll}2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2\end{array}\right)\left(\begin{array}{l}1 \\ 1 \\ 1 \\ 2\end{array}\right)=\left(\begin{array}{c}2 \cdot 1+1 \cdot 1 \\ 1 \cdot 1+2 \cdot 1+1 \cdot 1 \\ 1 \cdot 1+2 \cdot 1+1 \cdot 2 \\ 1 \cdot 1+2 \cdot 2\end{array}\right)=\left(\begin{array}{l}3 \\ 4 \\ 5 \\ 5\end{array}\right)$
10. (a) $A \vec{x}=2\left(\begin{array}{c}1 \\ -2 \\ -4\end{array}\right)+2\left(\begin{array}{l}2 \\ 3 \\ 1\end{array}\right)+3\left(\begin{array}{l}4 \\ 1 \\ 2\end{array}\right)=\left(\begin{array}{c}16 \\ 5 \\ 0\end{array}\right)$
(b) $A \vec{x}=1\left(\begin{array}{l}2 \\ 1 \\ 0 \\ 0\end{array}\right)+1\left(\begin{array}{l}1 \\ 2 \\ 1 \\ 0\end{array}\right)+1\left(\begin{array}{l}0 \\ 1 \\ 2 \\ 1\end{array}\right)+2\left(\begin{array}{l}0 \\ 0 \\ 1 \\ 2\end{array}\right)=\left(\begin{array}{l}3 \\ 4 \\ 5 \\ 5\end{array}\right)$
11. (a) $I=\left[\begin{array}{ll}1 & 0 \\ 0 & 1\end{array}\right]$
(b) $P=\left[\begin{array}{ll}0 & 1 \\ 1 & 0\end{array}\right]$
12. (a) $R=\left[\begin{array}{cc}0 & 1 \\ -1 & 0\end{array}\right]$
(b) $R^{2}=\left[\begin{array}{cc}-1 & 0 \\ 0 & -1\end{array}\right]$
13. $P=\left[\begin{array}{lll}0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0\end{array}\right], Q=\left[\begin{array}{lll}0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0\end{array}\right]$
14. $E=\left[\begin{array}{cc}1 & 0 \\ -1 & 1\end{array}\right], E=\left[\begin{array}{ccc}1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1\end{array}\right]$
15. $E=\left[\begin{array}{lll}1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1\end{array}\right], E^{-1}=\left[\begin{array}{ccc}1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1\end{array}\right] . E\left[\begin{array}{l}3 \\ 4 \\ 5\end{array}\right]=\left[\begin{array}{l}3 \\ 4 \\ 8\end{array}\right]$, and $E^{-1}\left[\begin{array}{l}3 \\ 4 \\ 8\end{array}\right]=\left[\begin{array}{l}3 \\ 4 \\ 5\end{array}\right]$.
16. $P_{1}=\left(\begin{array}{ll}1 & 0 \\ 0 & 0\end{array}\right), P_{1}=\left(\begin{array}{ll}0 & 0 \\ 0 & 1\end{array}\right) . P_{1}\binom{5}{7}=\binom{5}{0}$, and $P_{2}\binom{5}{0}=\binom{0}{0}$.
17. $\vec{w}=5 \vec{u}+7 \vec{w}$. Then $A \vec{w}=A(5 \vec{u}+7 \vec{v})=A(5 \vec{u})+A(7 \vec{v})=5 A \vec{u}+7 A \vec{v}$. Note for later in the semester: this isn't that interesting to note for the standard basis as given here, but it's interesting that given the action of a matrix on any set of vectors, we can determine its action on any vector spanned by them.

## Page 53

1. $l_{21}=5$

$$
\text { (2) } \begin{aligned}
x \quad+3 y & =1 \\
-6 y & =6
\end{aligned}
$$

2. $y=-1$, so $2 x-3=1$. Therefore $x=2$. Note that $2 \times(2,10)-(3,9)=(1,11)$. If the right hand side changes to $(4,44), x=8$, and $y=-4$.
3. $l=\frac{c}{a}$. The second pivot is $d-\frac{b c}{a}$, so we get:

$$
\begin{aligned}
a x \quad+b y & =f \\
+\left(d-\frac{b c}{a}\right) y & =g-\frac{c f}{a}
\end{aligned}
$$

So $y=\frac{a g-c f}{a d-b c}$.
5. If (e.g.) the second RHS is 2, there is no solution (any number other than 20 will do the trick). If the second RHS is 20 , there are infinitely many solutions. For example, $x=0$, $y=5$, and $y=0, x=\frac{10}{3}$.
7. If $a=0$, elimination breaks down temporarily, so swap the two equations:

$$
\begin{aligned}
4 x+6 y & =6 \\
+3 y & =-3
\end{aligned}
$$

So $y=-1$, which gives $4 x-6=6$, or $x=3$.

If $a=2$, we get a permanent breakdown:

$$
\begin{aligned}
\text { (2) } x+3 y & =-3 \\
4 x+6 y & =6 \\
\text { (2) } x+3 y & =-3 \\
+0 & =12
\end{aligned}
$$

13. 

$$
\begin{array}{r}
\text { (2) } x-3 y=3 \\
4 x-5 y+z=7 \\
2 x-y-3 z=5
\end{array}
$$

$$
\begin{aligned}
(2) x-3 y & =3 \\
+(1) y+z & =1 \\
+2 y-3 z & =2
\end{aligned}
$$

$$
\begin{aligned}
(2) x-3 y & =3 \\
+(1) y+z & =1 \\
-5) & =0
\end{aligned}
$$

So $z=0$, giving $y+0=1$, or $y=1$, giving $2 x-3=3$, or $x=3$.
17. Equal rows:

$$
\begin{aligned}
(2) x-y+z & =0 \\
2 x-y+z & =0 \\
4 x+y+z & =2
\end{aligned}
$$

$$
\begin{array}{r}
(2) x-y+z=0 \\
0+0+0=0 \\
+3 y-z=2
\end{array}
$$

$$
\begin{aligned}
(2) x-y \quad+z & =0 \\
+(3) y-z & =2 \\
0+0 \quad+0 & =0
\end{aligned}
$$

Nothing more to do!

Equal columns:

$$
\begin{array}{r}
2 x+2 y+z=0 \\
4 x+4 y+z=0 \\
6 x+6 y+z=2
\end{array}
$$

$$
\text { (2) } \begin{aligned}
\text { ( } x+2 y+z & =0 \\
+-1) z & =0 \\
+\quad-2 z & =2
\end{aligned}
$$

$$
\text { (2) } \begin{array}{rlrl}
\text { ( } x+2 y+z & =0 \\
+ & -1) & =0 \\
+\quad 0 & =2
\end{array}
$$

Third pivot is missing!

## Page 77

1. $B A=\left(\begin{array}{lllll}3 & 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 & 3\end{array}\right) ; A B=\left(\begin{array}{ccc}5 & 5 & 5 \\ 5 & 5 & 5 \\ 5 & 5 & 5\end{array}\right) ; A B D=\left(\begin{array}{l}15 \\ 15 \\ 15\end{array}\right) ; D C$ is not allowed, nor is $A(B+C)$.
2. (a) The matrix $A$ times the second column of $B$.
(b) The first row of $A$ times the matrix $B$.
(c) The third row of $A$ dot product with the fifth row of $B$.
(d) This is either:

- The first row of $C$ dot the first column of $D E$, which is the first row of $C$ dot the matrix $D$ multiplied by the first column of $E$; or
- The first row of $C D$ dot the first column of $E$, which is the first row of $C$ multiplied by the matrix $D$, dot the first column of $E$.

5. If $A=\left(\begin{array}{ll}1 & b \\ 0 & 1\end{array}\right)$, then $A^{2}=\left(\begin{array}{cc}1 & 2 b \\ 0 & 1\end{array}\right)$, and $A^{3}=\left(\begin{array}{cc}1 & 3 b \\ 0 & 1\end{array}\right)$. Also $A^{5}=\left(\begin{array}{cc}1 & 5 b \\ 0 & 1\end{array}\right)$, and $A^{n}=\left(\begin{array}{cc}1 & n b \\ 0 & 1\end{array}\right)$.

If $A=\left(\begin{array}{ll}2 & 2 \\ 0 & 0\end{array}\right)$, then $A^{2}=\left(\begin{array}{ll}4 & 4 \\ 0 & 0\end{array}\right)$, and $A^{3}=\left(\begin{array}{ll}8 & 8 \\ 0 & 0\end{array}\right)$. Also $A^{5}=\left(\begin{array}{cc}32 & 32 \\ 0 & 0\end{array}\right)$, and $A^{n}=\left(\begin{array}{cc}2^{n} & 2^{n} \\ 0 & 0\end{array}\right)$.
6. $(A+B)^{2}=\left(\begin{array}{cc}10 & 4 \\ 6 & 6\end{array}\right) ; A^{2}+2 A B+B^{2}=\left(\begin{array}{cc}16 & 2 \\ 3 & 0\end{array}\right) \cdot(A+B)(A+B)=A^{2}+A B+B A+B^{2}$.
26. $A B=\left(\begin{array}{ll}1 & 0 \\ 2 & 4 \\ 2 & 1\end{array}\right)\left(\begin{array}{lll}3 & 3 & 0 \\ 1 & 2 & 1\end{array}\right)=\left(\begin{array}{l}1 \\ 2 \\ 3\end{array}\right)\left(\begin{array}{lll}3 & 3 & 0\end{array}\right)+\left(\begin{array}{l}0 \\ 4 \\ 1\end{array}\right)\left(\begin{array}{lll}1 & 2 & 1\end{array}\right)=\left(\begin{array}{lll}3 & 3 & 0 \\ 6 & 6 & 0 \\ 6 & 6 & 0\end{array}\right)+\left(\begin{array}{lll}0 & 0 & 0 \\ 4 & 8 & 4 \\ 1 & 2 & 1\end{array}\right)=$ $\left(\begin{array}{ccc}3 & 3 & 0 \\ 10 & 14 & 4 \\ 7 & 8 & 1\end{array}\right)$

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4. If (e.g.) $A=\left(\begin{array}{ll}0 & 1 \\ 0 & 0\end{array}\right)$, then $A^{2}=0$. However, since the $(i, j)$ entry of $A^{T} A$ is the dot product of the $i^{\text {th }}$ column of $A$ with the $j^{\text {th }}$ column of $A$, the only way all entries could be 0 is if all the columns have length 0 . Otherwise there will be at least one positive entry on the diagonal (since the diagonal entries are the squares of the lengths of the corresponding columns.)

## Additional Problem

1. $A \cap B$ is a line.
2. Any linear combination of the two equations will do. For example, just adding the two gives $C$ as the plane $2 x-y=1$.
3. Adding the two planes, but changing the constant will do the trick. So, for example, taking $D$ to be the plane $2 x-y=10$ has $A \cap B \cap D$ empty, but $A \cap B$ and $B \cap D$ are both lines. The three lines are parallel, so there is no intersection.
