Big Assignment 1

- This assignment covers work from Day 1-1 to Day 4-2.

- You have ten days to complete the assignment. It is due Sept 24 at 8am.

- You may use the worksheets, labs, and your notes, as well as any other resources.

- However, you may not work with anyone else on this assignment – it must be entirely your own work.

- Throughout the assignment, show your work so that your reasoning is clear. Otherwise no credit will be given. Circle your answers. For example, copying graphs from graphing software is insufficient ‘work’.

- Do not spend too much time on any one problem. Read them all through first and attack them in an order that allows you to make the most progress.

- You must hand in a scanned version of this assignment on Gradescope and no other pages. If you need to do rough work, do it elsewhere before writing your answers.

- The number of points on each question is shown next to the question number. There are 111 points available in total, plus four bonus points.

- You must sign the statement at the bottom of this page.

Honor Code:

I have adhered to the Duke Community Standard and all the rules above in completing this assignment.

Signature: ____________________________
1. (12 Points) Consider the graphs, tables, or expressions below. For each, either write down a specific reason it is not a function, or (if it is a function), write down its range. In each case, the complete domain is shown.

A. \( \sqrt{x}, \ x \geq 0 \).

B. \[
\begin{array}{cccc}
 x & 1 & 2 & 3 & 4 \\
 y & 0 & 2 & 0 & 3 \\
\end{array}
\]

C. 

D. \( \sqrt{x-2}, \ x \geq 0 \).

E. \[
\begin{array}{cccc}
 x & 1 & 2 & 3 & 3 \\
 y & 0 & 2 & 1 & 3 \\
\end{array}
\]

F. 

A.

B.

C.

D.

E.

F.
2. (18 Points) The number of pens under a student’s desk \( t \) days after the start of the semester is well modeled by the function \( P(t) = 1.5t + 2 \).

(a) Fill in the blanks: On average, the student drops ______ _______ per _______ under their desk. 2

(b) What does the number 2 represent in this model?

(c) Suppose the number of pens under the student’s desk has increased by 18 pens. How many days have passed?

(d) Suppose that at the end of the 20\textsuperscript{th} day, the student cleans up all the pens from under their desk. Suppose also that the student then continues dropping pens at an average rate of one every two days for the rest of the semester, which is 100 days long in total. Write down a piecewise function for \( P(t) \), the number of pens under the students desk at time \( t \) after the start of the semester.

\[
P(t) =
\]

(e) What is \( \lim_{t \to 20^+} P(t) \)? Show your work.

(f) Is \( P(t) \) a continuous function? Explain your answer.
3. (12 Points) Match the following formulas on the left to their graphs on the right. There may be formulas that do not correspond to any graph, and there may be graphs that do not correspond to any formula. All non-zero horizontal and vertical asymptotes are shown. Graphs may not all have the same scales.

A. \( x^2(x - 1) \)

B. \( x^2(x - 1)^2 \)

C. \( x^3(x - 1)^3 \)

D. \( \frac{x^2}{(x-1)^2} \)

E. \( \frac{x^2}{(x-1)^3} \)

F. \( \frac{(x-1)^3}{x^2} \)

A. _____  B. _____  C. _____  D. _____  E. _____  F. _____
4. (12 Points) Consider the following graph of a function \( f(x) \). On the axes below, draw the graph of \( 2f(-2x - 2) \). Label the \( x \) and \( y \)-intercepts. Feel free to use scratch paper to draw intermediate steps, but only draw the final result below. In addition, write down each of the transformations you used (e.g. ‘shifted up by 4 units’). Write them down in the order you did them on scratch paper.
5. (18 Points) Consider the function \( f(x) = \frac{x^3+3x^2+3x+2}{x+2} \).

(a) Show that if you attempt to compute \( f(-2) \), you get an indeterminate form.

(b) By using a table method, estimate the value of \( \lim_{x \to -2} f(x) \).

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So it seems like the limit is ____.

(c) Fill in the blank: In Part (a), when you plugged \( x = -2 \) into the numerator, you got zero. In other words, \( x = -2 \) is a root of \( x^3 + 3x^2 + 3x + 2 \). This means that \( (x + \text{____}) \) is a factor of the numerator.

(d) The other factor of the numerator is \( x^2 + x + 1 \). Use this to factor the numerator and compute the limit in part (b) algebraically. Carefully show all your steps and use correct notation.

(e) Note that \( x^2 + x + 1 = (x + \frac{1}{2})^2 + \frac{3}{4} \). Use this to draw the graph of \( f(x) \), labeling its vertex's coordinates precisely and explaining how you found them (No calculus allowed here! Hint: function transformations!). Take care at \( x = -2 \!).
6. (17 Points) Consider the function \( f(x) = x^2 + x \). A graph of this function is drawn below:

(a) Compute the average rate of change of \( f(x) \) between \( x = -1 \) and \( x = 1 \).

(b) Draw a line on the graph whose slope represents your computation and answer in part (a).

(c) Using a table method, estimate the instantaneous rate of change of the function at \( x = -1 \). That is, compute the average rates of change of \( f(x) \) on \([-1, -1 + h]\) for small values of \( h \), and use these to estimate the instantaneous rate of change of \( f(x) \) at \( x = -1 \).

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It seems that the instantaneous rate of change of \( f(x) \) at \( x = -1 \) is \( \) .

(d) Use the limit definition of instantaneous rate of change to compute the instantaneous rate of change of the function at \( x = -1 \) (i.e. compute \( f'(-1) \)).

(e) Draw a line on the graph whose slope represents the instantaneous rate of change you estimated and computed in the last two questions. Be sure to label both this line and the line from part (b) to let me know which is which!
7. (12 Points) A function is *continuous at* \( x = a \) if \( \lim_{x \to a} f(x) = f(a) \). There are three ways that we studied in which this can go wrong. Draw an example of each, and explain why each of your examples fails the definition.

(a) \( \lim_{x \to a} f(x) \) does not exist.

(b) \( \lim_{x \to a} f(x) \) exists, but is not equal to \( f(a) \).

(c) \( \lim_{x \to a} f(x) \) exists, but is infinite.
8. (10 Points) Each of the following statements is false in general (though may be true for particular instances). For each, briefly explain why it is false in general.

(a) If an object’s velocity is decreasing, then the object is slowing down.

(b) Using a linear trendline is a good way to find a model for data.

(c) If an object is accelerating (in the colloquial sense), then its acceleration is positive.

(d) The slope of a function can be computed using the formula \( \frac{y_2 - y_1}{x_2 - x_1} \) for any two points \((x_1, y_1)\) and \((x_2, y_2)\) on its graph.

(e) Consider an object is moving under the force of gravity. If its measured height at \(t = 0.03\) seconds the same as its measured height at \(t = 0\) seconds, we can conclude that the object was dropped, rather than thrown up or down.
**Bonus Question (2 points)** There are municipal elections coming up in Durham. Research the web presence of the two leading candidates for Mayor, Javiera Caballero and Elaine O’Neal. Write down two substantial policy differences between the two candidates.

**Bonus Question (2 points)** Draw a funny picture, paste in a good meme, tell me a good story, or tell me a good joke.