Exercise 7.1, pg 366

2e.

Let

\[ A = \begin{pmatrix} 3 & -2 & 0 & 1 \\ 2 & 3 & 1 & 0 \\ 0 & 1 & 3 & 2 \\ 1 & 0 & -2 & 3 \end{pmatrix}. \]

Grinding through the characteristic polynomial (to make life a little easier you can swap rows 1 and 4, and rows 2 and 3; and then use 5.2.10b), we get

\[ p_A(t) = t^4 - 12t^3 + 60t^2 - 144t + 160. \]

A flash of inspiration yields

\[ p_A(t) = (t^2 - 4t + 8)(t^2 - 8t + 20). \]

This implies that the eigenvalues are \( 4 \pm 2i \) and \( 2 \pm 2i \). The four corresponding eigenvectors give a basis which diagonalizes the matrix. Let \( P \) be the matrix whose columns are these eigenvectors:

\[ P = \begin{pmatrix} 1 & 1 & -1 & -1 \\ -i & i & i & -i \\ -i & i & -i & i \\ 1 & 1 & 1 & 1 \end{pmatrix}. \]

Thus \( A = PJP^{-1} \), where

\[ J = \begin{pmatrix} 4 + 2i & 4 - 2i & 2 + 2i & 2 - 2i \\ 4 - 2i & 2 + 2i & 2 - 2i \\ 2 + 2i & 2 - 2i & \end{pmatrix}. \]

By question 1 we also construct a basis giving the columns of \( C \):

\[ C = \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}. \]

such that \( A = CDC^{-1} \) where

\[ D = \begin{pmatrix} 4 & -2 & 0 & 0 \\ 2 & 4 & 0 & 0 \\ 0 & 0 & 2 & -2 \\ 0 & 0 & 2 & 2 \end{pmatrix}. \]