Curriculum Vitae

Arlie O. Petters, MBE
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Research Interests

- *Mathematical Physics*:
  Mathematics—tools from differential geometry, singularities, and probability theory
  Physics—problems connected to the interplay of gravity and light (gravitational lensing, general relativity, astrophysics, cosmology)
- *Mathematical and Scientific Methods in Business Administration:*
  Mathematical finance with applications
  Entrepreneurship and innovation in STEM[1] fields (developing world)

Professional History[2]

- *Dean of Academic Affairs for Trinity College of Arts & Sciences* and *Associate Vice Provost of Undergraduate Education*, Duke University (2016–present)
- *Chairman*, Council of Science Advisers to the Prime Minister of Belize (2010–2013)
- *Benjamin Powell Professor of Mathematics*, Duke University (2009–present)
  –*Professor of Economics*,
  Duke University (2016–present)
  –*Professor of Business Administration*,
  Fuqua School of Business, Duke University (2008–2017)
  –*Professor of Mathematics and Physics*, Duke University (2003–present)
- *Martin Luther King Jr. Visiting Professor of Physics*,
  MIT (2003–2004)
- *President and Founder*, Petters Research Institute,
  Dangriga, Belize (2005–present)
- *William & Sue Gross Associate Professor of Mathematics*,
  Duke University (1998–2003)
- *Director of Undergraduate Studies*, Mathematics,
  Duke University (2002)
- *Assistant Professor of Mathematics*,
  Princeton University (1993–1998)
- *Co-Director of Graduate Studies*, Mathematics,
  Princeton University (1996–1998)

[1]STEM: Science, Technology, Engineering, and Mathematics.
[2]A list of my administrative/service experience is given in Section 3.2 (page 44).

- *Instructor of Pure Math*, MIT (1991–1993)
- *Member of Technical Staff*, Bell Laboratories, Murray Hill, NJ (Summers, 1986–1990)
- *Visiting Professor*, Princeton University (2007), MIT (2003–2004),
  Harvard University (2002), Oxford University (1995),
  Max-Planck-Institut für Astrophysik (1994)

Education

- *Ph.D. MIT*, 1991 (Mathematics)
  –Ph.D. thesis advisers: Bertram Kostant (MIT) and
  David Spergel (Princeton University)
  Ph.D. thesis title: *Singularities in Gravitational Microlensing*
  –Princeton University, 1988–1991, Department of Physics
  (Exchange Scholar; in absentia from MIT)
  –MIT, 1986-1988, Department of Mathematics
- *B.A./M.A. Hunter College*, 1986 (Mathematics and Physics)
  M.A. thesis title: *The Mathematical Theory of General Relativity*

Awards and Honors

- Selected as Grand Marshal of the 2012 COFECA Central American Parade in L.A., received certificate of recognition from the mayor of L.A. (2012)
- Caribbean American Heritage Award for Excellence in Science and Technology, The Institute of Caribbean Studies, Washington, D.C. (2011)
- Robert L. Clark Award for Scientific Achievement, Bronx Community College of the City University of New York (2011)
- Appointed by the Prime Minister of Belize as Chairman of his inaugural Council of Science Advisers, Belize (2010–present)
- Honored with street name, *Dr. Arlie Petters Street*, Dangriga, Belize (2009)
- Named by the Queen of England to Membership in the Most Excellent Order of the British Empire (MBE, 2008)
- Honorary Doctor of Science, Hunter College of the City University of New York (2008)
- Award for Service to Dangriga and Belize (2007), The New York City Garifuna Community Association
- Award for Service to the Educational Development of Belize (2007), Friends in Support of the Diocese of Belize, New York
- Portrait Inductee, National Academy of Sciences Portrait Gallery of Distinguished African-American Scientists and Engineers (2006)
- NSF Mathematical Innovations Grant Award (2004–2008)
- Blackwell-Tapia Prize in Mathematical Science (first recipient, 2002)
- Inductee, Bass Society of Fellows, Duke University (1998–present)
- NSF Faculty Early Career Grant Award (1998–2003)
- Alfred P. Sloan Research Fellowship (1998–2002)
- Hall of Fame Inductee, Hunter College of the City University of New York (1999)
- Belizeans in Solidarity Award for Outstanding Academic Achievement (1996)
- Award for Service to African-American Students at Princeton University, Princetonians of Color Network (1996)
- Sigma Xi (Full Member, Elected 1992, MIT Chapter)
- Bell Laboratories CRFP Fellow (1986–1991)

- Harold Hoey, Jr. Scholarship Award in Mathematics (1986)
- Gillet Alumni Prize in Physics (1986)
- Joseph A. Gillet Memorial Prize in Physics (1986)
- Rainer Sachs Scholarship in Mathematics (1986)
- Minority Access to Research Careers Fellow (1983–1985)

Boards

- Board of Trustees, Institute of Pure and Applied Mathematics (2006–2010)
- Board of Governors, Institute for Mathematics and its Applications (2006–2010)

Professional Societies

- Fellow, The American Mathematical Society (2013-present)
- Fellow, The Royal Astronomical Society (2010-present)
- Bass Society of Fellows, Duke University (1998-present)
- Renaissance Weekend

Personal Information

- Date/Place of Birth: February 8, 1964, Belize
- Citizenship: U.S.A. (1990)
- Status: married

CONTENTS

1 Research Experience

In this section, an expression of the form "[x]" cites the corresponding entry in the list of publications in Section 1.3 (page 24).

1.1 Research Overview

My research deals with the mathematical and physical aspects of *gravitational lensing*, which is the action of gravity on light. In a typical gravitational lensing scenario, light originates from a distant source like a star, galaxy, or quasar, and experiences deflection by the gravity of a foreground mass before arriving on earth. The deflector could be a star, galaxy, clump of dark matter, or even a black hole.

The earliest scientific papers on gravitational lensing date back to at least the early 1780s, when Michel, Laplace, and Cavendish applied Newton's theory of gravity to lensing. However, it was not until 1915 that Einstein, using his brand-new gravitational theory—the General Theory of Relativity—derived the correct formula for how much gravity bends light. His prediction for the bending angle of starlight grazing the sun, which is twice the angle obtained via Newton's gravitational theory, was confirmed in 1919 by Sir Arthur Eddington (within the experimental accuracy of the time). This marked the first observed example of gravitational lensing. The turning point in the subject, though, was triggered by the 1979 discovery of lensing outside our solar system. Since then hundreds of examples of extra-solar gravitational lensing events have been observed and the field has undergone exponential growth.

Today, gravitational lensing attracts observational astronomers, astrophysicists, mathematical physicists, and mathematicians across the globe. Lensing is exciting because it can address the nature of dark matter, the existence of extra-solar planets, the existence of black holes, the existence of a possible extra dimension of physical space, etc. My work straddles and creates synergistic interactions between the mathematical and physical aspects of the subject.

Research Period: 1991–2001

Development of a Mathematical Theory of Weak-Deflection Gravitational Lensing

Why Develop a Mathematical Theory of Gravitational Lensing? Astrophysicists typically infer physical properties like the mass composition and cosmological parameters of the universe using heuristic physical arguments, intuition gained from numerical simulations, and models based on idealized analytical forms, causing some theorists to oversimplify the richness and complexity of real systems. In addition, prior to 1991 most lensing research focused only on the situation of a single deflector between the source and observer, so little was known about the multiple lens plane case. This begged for a theory that yields the *universal* properties of multi-plane lens systems, namely, those features that are *generic* among lens models and *stable* against small perturbations of the models. Such universal properties would be essentially independent of the details of a chosen lens model and provide important insights into the intrinsic physical features of realistic lens systems.

To accomplish this ambitious feat, one has to develop a mathematical theory of genericity and stability for general multi-plane gravitational lens systems. This is because astrophysics serves as a guide to understanding *specific examples* of gravitational lensing, but it is the mathematics that enables the creation of a general theory applicable to *essentially all* lens systems. The desired theory would put the field on a solid mathematical foundation and simultaneously solve a string of core theoretical lensing problems. A further important benefit is that such a mathematical theory would transcend gravitational lensing. In other words, the theory becomes applicable to any real-world setting invoking a mathematical framework similar to the one in lensing. For example, the caustics of gravitational lensing appear not only in that subject, but also in fields ranging from industrial robotics to oil exploration. Hence, a mathematical theory of gravitational lensing not only provides tools for the field of lensing, but also has the potential to be applicable to several areas outside the subject.

Overview of Papers [28] to [52]

From 1991 to 2001, the research in papers [28] to [52] was focused primarily on developing a mathematical theory of weak-deflection gravitational lensing through solving several key theoretical lensing problems. This required an unlocking of the mathematical theorems and structures governing the core of gravitational lensing, namely, *images* (multiple images, image magnification, image time delays) and *caustics* (local and global properties).

- **Images**
  - **Image Counting Problem**
    -A striking feature of gravitational lensing is the formation of multiple images of a background source. The Image Counting Problem is to determine counting information about the number of images produced by general deflectors distributed over a finite number of lens planes. The problem was solved in the papers [44, 50–52] using Morse theory. The basic strategy was to convert the Image Counting Problem into counting critical points of a non-degenerate function. The papers showed that all the Betti numbers of the domains of the appropriate non-degenerate functions can be computed for single and multi-plane lensing, giving a general set of counting formulas and lower bounds on the number of lensed images. The results included an odd-number-image theorem for multi-plane nonsingular lensing.
    -An upper bound on the number of images in the multi-plane point mass case was found in paper [36]. The latter paper used the theory of resultants, rather than Morse theory, to find an upper bound. The paper also recovered the single-plane result and provided a rigorous proof for that case. In paper [33], Mao, Witt, and I made a conjecture about the maximum number of images due to point mass lenses: the maximum number $N_{\max}$ of lensed images is linear (rather than quadratic) in the number g of point masses. The conjecture was shown to hold.[3]
  - **Fixed-Point Images, Image Magnification, and Image Time Delays**
    -When a source is gravitationally lensed, its images are shifted from the true position of the source. A *fixed-point image* is an angular image position that corresponds to the original angular source position, despite gravitational lensing. Do such images exist and, if so, how many are there? Papers [31, 32] introduced the notion of fixed-point images in gravitational lensing, established the existence of such images, and used Morse theory and the theory of resultants to find counting formulas and bounds for the number of

[3] In 1997, we established our conjecture for $g = 3$ [33]. For $g \geq 4$, Khavinson and Neumann proved in 2004 that $N_{\max} \leq 5g - 5$, which was shown to be attainable by an argument of Rhie (2003) and then by Bayer and Dyer (2007) using a more physical approach in terms of an upper bound on the mass.

these images. Paper [31] also established an odd-number theorem for the fixed-point images due to nonsingular isolated deflectors. For three-point masses, the paper shows that there are two fixed points and they lie at the foci of an ellipse inscribed in the triangle defined by the positions of the three masses. Each image of a lensed source has a magnification. Paper [37] uses Morse theory to determine lower bounds on the total magnification due to a generic single-plane gravitational lens. The lower bounds are given in terms of the number of images of the source, number of obstruction points of the deflector, and mass density of the deflector. The paper includes a treatment of the magnifications and trajectories of the images as the source moves to infinity.
-Paper [49] was the first to formulate gravitational lensing from a symplectic geometric viewpoint using Arnold's singularity theory. The latter was employed to give a local classification of the image surfaces and time-delay surfaces for a generic gravitational lens system. This includes a local classification of the lens system's Maxwell set (source positions where at least two images have the same time delay), which defines the boundary between regions where all the images have distinct time delays. (The time delay between images of a lensed quasar is important as the delays are connected with the age of the universe.)

- **Caustics**

  Caustics play a central role in the theory of gravitational lensing. They are the places where a source appears the brightest, meaning positions from which a source has at least one formally infinitely magnified image. The local and global geometry of caustics were explored in papers [25, 28, 29, 30, 34, 35–43, 45–49].

  - **Local Geometry of Caustics**
    -Paper [49] reformulated gravitational lensing from the viewpoint of Arnold's singularity theory. This resulted in a local classification of the caustics, caustic evolution (metamorphoses), big caustics, and bi-caustics (curves traced out by cusps) for a generic gravitational lens system. Some additional caustic phenomena, such as handkerchief singularities, were also explored in [45].
    -Paper [30] was the first to characterize the qualitative local behavior of the center-of-light or centroid of a source crossing the caustics due to a generic gravitational lens. The primary mathematical tool employed was Whitney's singularity theory.

  - **Global Geometry of Caustics**
    -Paper [43] gave a global mathematical law connecting the curvature of caustics to obstruction points of the lens (opacity of the lens). The paper also determined lower bounds on the number of obstruction points, upper bounds for the total curvature of the caustics, and a cusp counting formula, which refines and expounds further on an earlier formula in [46].
    -Paper [41] determined an upper bound on the total number of cusps in a caustic network due to point masses on a plane with continuous matter and shear and showed that all the cusps can be eliminated if the density of continuous matter is sufficiently large. The paper also determined the global equations governing bi-caustics.

    -Papers [40, 42] discovered new caustic phenomena in double plane lensing, with each plane having a point mass with continuous matter and shear. Some of these new geometric caustic forms include teardrop, cardioid, and kidney-bean caustics. The paper shows how certain caustics in double-plane lensing cannot occur in the single-plane case. For

example, all five generic caustic metamorphoses—lips, beak-to-beak, swallowtails, elliptic umbilics, and hyperbolic umbilics—occur in double-plane point mass lensing, while lips and hyperbolic umbilics are impossible for the single-plane point mass case.
-Paper [47] gave an extensive classification of the number caustics and cusps due to one- and two-point mass lenses on a plane with continuous matter and shear. The paper also proves that g-point masses with continuous matter and shear can have at most $3g-3$ beak-to-beak caustic singularities.
-Papers [28, 35, 39, 38, 45] made the first steps towards a unified, coherent theory of the many global results in [43, 41, 40, 42, 47].

- **Wavefronts, Caustic Surfaces and Caustic Surfing**
Can gravitational lensing be studied from a wavefront perspective? Yes. The source emits a wavefront that is initially spherical, but which becomes distorted and develops singularities as it passes through the lens. The caustics on the wavefront then trace out surfaces in space as the wavefront moves. These caustic surfaces are extensions of the planar caustic curves studied earlier in papers [28, 30, 34, 35–43, 45–49]. Paper [29] briefly investigated the wavefronts and caustic surfaces due to a lens with an elliptical potential (e.g., galaxy lens). This work was extended significantly in paper [25], which derived the equations governing wavefronts and caustics on wavefronts, and gave detailed illustrations of the caustics and wavefronts in space. An interesting application of the work is to caustic surfing, a futuristic notion suggested by Roger Blandford in his 2001 Millennium Essay. By surfing a caustic surface, a space-borne telescope can be fixed on a gravitationally-lensed source to obtain observations of the source at an extremely high magnification over an extended time period. Such observations would reveal the structural nature of distant sources like quasars that could not otherwise be resolved. Paper [25] presents a way to surf caustic sheets due to a lensed source in rectilinear motion. This paper also used wavefronts to explain a puzzling effect that was first mentioned in the book [2, p. 188]. Specifically, for a singular isothermal sphere lens there is a simple closed non-caustic curve that separates the plane of the source into two regions where the number of images differs by one, rather than the expected number of two. We showed that such a curve is the boundary of the associated wavefront, and the number of images differs by one because a whole sheet of the wavefront is missing due to the obstruction point of the singular isothermal sphere.

Summary of the Monograph [2]

The work in papers [28] to [52] culminated in the monograph [2] by me (lead author), Levine, and Wambsganss. The book refined some of the findings in those papers and included new results. It gave a comprehensive and detailed exposition of the mathematical theory of gravitational lensing.

The book [2] employed singularity theory from pure mathematics not only to place gravitational lensing on a unified mathematical foundation, but also to show in great detail how the mathematical framework can be employed to solve certain key lensing problems that were impenetrable using non-singular-theoretic tools. For example, the book gave detailed solutions for the following problems for k-plane lensing: classify the stable properties of time delay families and lensing maps using an astrophysically natural notion of equivalence and establish the genericity of stable properties among all lens systems (Chapter 8); characterize the generic local qualitative properties of critical curves, caustics, and their metamorphoses (Chapter 9); determine invariant counting formulas for the number of images (Chapter 10–12); characterize the behavior of individual image magnifications, determine lower bounds on the total magnification, and establish a global magnification cross section formula and its scalings near caustics (Chapters 9, 13); and determine a formula relating the curvature of caustics with the opacity of a lens (Chapter 15). On the other hand, the astrophysics of gravitational lensing also came to bear on singularity theory. For example, lensing issues led us to develop a theory of stability and genericity for families of time delay functions and lensing maps for k-plane lensing, generalize Thom's fundamental catastrophe theorem for two control parameters, link the curvature of caustics curves to topological invariants of the lens, create a theory of local convexity of caustics, and introduce probability theory into singularity theory via magnification cross sections (Chapters 8, 9, 13, 15). Overall, the book established a cross-fertilization spanning the spectrum from highly abstract mathematical methods to practical astrophysical theory pertinent to observational lensing data.

It is important to add that singularity theory is a collection of many theories. We employed five of these theories in the book: Morse theory, Whitney singularity theory, Thom catastrophe theory, Mather stability theory, and Arnold singularity theory. We created a seamless joining of all these theories in order to have a single coherent theory that is applicable to gravitational lensing—a highly nontrivial task! This *synthesis* involved establishing the equivalence of certain concepts and results from the different singularity theories and included retooling certain key definitions and theorems as well as introducing new ones (Chapters 7–10, 14, 15). For example, we established the equivalence of seven different ways of representing fold and cusp caustics in the various theories; introduced a fold-cusp function as a tool in characterizing the approximate *quantitative* behavior of lensing near caustics; adopted the notion of genericity to the space of families of time delay functions; and dissected each concept of differentiable equivalence (at least ten) to determine those that are consistent with the physics of lensing. Furthermore, given the theoretically hard and technical nature of singularity theory, we had to translate many abstract results—most of which involved calculations with abstract mathematical spaces (singularity manifolds, multijet bundles, etc.) rather than numbers or variables—into practical formulas that can be used to compute explicitly the key physical quantities of a lens system (e.g., lens mass, lens distance, source radius).

Overall, the core techniques and theorems presented in the book [2] can be used to make explicit predictions about lensing behavior accessible to current and near-future instrumentation. In addition, the book provides a platform from which to generalize weak-deflection gravitational lensing to the strong deflection case (e.g., lensing by black holes). These aspects were addressed in the research papers that came after completing the book in June of 2001 and are taken up in the next subsection.

Research Period: 2002–2007

Predictions About Center-of-Light

Employing some of the results from the mathematical theory of lensing [28]–[52], papers [26] and [27] advanced the theory of the center-of-light curves (or image centroids) due to caustic crossings in a generic setting. This resulted in the characterization of those curves and their predicted shapes, which should be accessible to the Space Interferometry Mission Planet Quest (SIM PlanetQuest) slated to be launched by NASA in 2015.

- **Center-of-Light for Sources Crossing Fold Caustics**
  Paper [27] explored microlensing near *fold* caustics. Microlensing describes gravitational lensing of a source whose multiple images are not resolved. The fundamental microlensing observables are the total magnification (photometry) and centroid (astrometry) of the images of a lensed source. Using some of the mathematical results from the book (especially Sections 9.2 and 9.3), we characterized in [27] the local behavior and derived analytic expressions for the photometric and astrometric behavior near a generic fold caustic for point and extended sources. Also, in paper [27] we *predicted* the detailed shape of the astrometric curve of an extended source (with uniform as well as limb darkening surface brightness profiles) crossing a fold caustic. The predicted curve has a generic S-like shape and should be accessible to the planned instrumentation for SIM PlanetQuest. Furthermore, we showed how the predicted astrometric curve can be used to infer the angular size of the source. Coupling photometric and astrometric data, we also illustrated how our equations can be used to determine the mass of the lens. The results in [27] can be applied to binary lenses in our galaxy and cosmological (quasar) microlensing.

- **Center-of-Light for Sources Crossing Cusp Caustics**
  Paper [26] investigated microlensing near *cusp* caustics by drawing on some of the results in Sections 9.2 and 9.3 of the book. We derived explicit formulas for the total magnification and centroid of the images created for sources outside, on, and inside a cusped caustic curve. We obtained new results on how magnification scales with respect to separation from the cusped caustic curve for arbitrary source positions. For example, along the axis of symmetry of the cusp, the magnification μ is proportional to u^{-1}, where u is the distance of the source from the cusp, whereas perpendicular to this axis, μ is proportional to $u^{-2/3}$. We also *predicted* that the generic shape of the image centroids for point and extended sources have "swallowtail" and "parabolic" features, respectively. These predicted features should be accessible to the instruments on SIM PlanetQuest. When a point-like source passes through a fold arc, the image centroid has a jump discontinuity. We derived a formula for the size of the image centroid jumps for the folds abutting a cusp and used the formula to outline a method by which the central parameter for microlensing (i.e., the angular Einstein ring radius) can be estimated using measurements of the jump. Similar to the fold case, the results in [26] are applicable to Galactic binary lenses and cosmological microlensing, which can yield estimates of the lens's mass and angular and angular radius of the source.

Predictions Identifying Small-Scale Structure in Galaxies Using Flux-Ratio Anomalies

The problems of what fraction of dark matter exists in galactic halos and the degree to which the dark matter is clumpy or smoothly distributed are indeed pressing in astronomy. Papers [22] and [23] also apply results from the mathematical theory of lensing [28]–[52], showing rigorously how flux-ratio anomalies of sources near fold and cusp caustics can be used to test, in the most generic way possible, for the presence of small-scale structure (like dark matter clumps) in galactic halos.

- **Identifying Small-Scale Structure Using Cusp Lenses**
  Paper [23] gives a more robust and generic method for identifying galactic gravitational lenses with small-scale structure (possibly dark matter clumps) using cusp caustic singularities. For a close triplet of images associated with a source near a cusp caustic, the sum of the signed magnifications should approximately vanish. We derive realistic upper bounds on the sum and argue that lenses with flux ratios that significantly violate the bounds can be said to have structure in the lens potential on scales smaller than the image separation. We predict that *three* of the observed cusp lenses have small-scale structure giving rise to flux ratio anomalies in their quad-images: B2045+265, B0712+472, and 1RXS J1131-1231.

- **Identifying Small-Scale Structure Using Fold Lenses**
  When the source in a 4-image gravitational lens lies close to a fold caustic, two of the lensed images lie close together. If the lens potential is smooth on the scale of the separation between the two close images, then the normalized difference $R_{\rm fold}$ between their fluxes should approximately vanish. Violations of this fold relation in observed lenses are thought to indicate the presence of structure (possibly dark matter clumps) on scales smaller than the separation between the close images. Paper [22] studies the fold relation and finds it to be more subtle and rich than was previously realized. The degree to which $R_{\rm fold}$ can differ from zero for realistic smooth lenses depends not only on the distance of the source from the caustic, but also on its location along the caustic, and on the angular structure of the lens potential. It is then impossible to say from $R_{\rm fold}$ alone whether observed flux ratios are anomalous or not. Instead, we must consider the full distribution of $R_{\rm fold}$ values that can be obtained from smooth potentials that reproduce the separation between the two close images and the distance to the next nearest image. Paper [22] analyzes the generic and specific features of this distribution, and then predicts that 5 of the 12 known lenses with fold configurations have flux ratio anomalies: B0712+472, SDSS 0924+0219, PG 1115+080, B1555+375, and B1933+503. Combining this with the results of paper [23], we conclude that at least half (8 out of 16) of all 4-image lenses (known in March of 2005) that admit generic, local analyses exhibit flux ratio anomalies indicative of small-scale structure.

Tests of Gravitational Theories Through Accessible Lensing Predictions

One of the fundamental goals of physics is to test whether Einstein's theory of gravity is correct on different scales. Papers [19–21, 24] present tests of general relativity and other gravitational models using gravitational lensing by compact objects like black holes and neutron stars.

- **Einstein's Theory of Gravity: Probing Black Holes**

  * **Schwarzschild Black Holes, Primary and Secondary Relativistic Images**

    Lensing by a black hole produces two outer images, called *primary images*, and a family of *secondary images* that loop around the black hole. The centroid and total magnification of these images provide a potential test of general relativity. Paper [24] determined an analytical framework for how the secondary relativistic images perturb the image centroid and total magnification of the primary images for point-like and extended sources with arbitrary surface brightness profiles. The results were applied to the massive black hole at the center of our galaxy, showing that a single factor characterizes the full relativistic secondary-image correction to the image centroid and total magnification. The paper also showed that as the lens-source distance increases, the relativistic correction factor strictly decreases and that the correction factor is minuscule, of order 10^{-14}. This demonstrates the importance of the primary images relative to the secondary ones in terms of the capabilities of near-future instrumentation.

  * **Spherically Symmetric Black Holes, Primary Images, and Cosmic Censorship**

    Paper [21] gives a higher-order asymptotic framework for testing general relativity using lensing by a spherically symmetric black hole. The paper gives an invariant series that goes beyond the standard weak-deflection bending angle term. We computed first- and second-order corrections to the primary image positions, magnifications, and time delays to lensing by a Schwarzschild black hole. For a Reissner-Nordstrom black hole, our formalism reveals an intriguing mathematical connection between lensing observables and the condition for having a naked singularity. This could provide an observational method for testing the existence of such objects and so impact the Cosmic Censorship Conjecture (which rules them out). This conjecture is one of the most important in General Relativity. We applied the analytical results of the paper to the Galactic black hole and *predict* that the corrections to the image positions are at the level of 10 micro-arcseconds, while the correction to the time delay is a few hundredths of a second. These corrections would be measurable today if a pulsar were found to be lensed by the Galactic black hole, and they should be readily detectable with proposed missions like MAXIM.

* Kerr Black Holes, Spin, and Cosmic Censorship

Paper [16] extends the weak-deflection work of [21, 20] from static spherically symmetric black holes to the non-static case, namely, Kerr black holes. A Kerr black hole with mass parameter m and angular momentum parameter a acting as a gravitational lens gives rise to two images in the weak field limit. Paper [16] derives the magnification relations, namely the signed and absolute magnification sums and the centroid up to post-Newtonian order for Kerr black holes. The paper shows that there are post-Newtonian corrections to the total absolute magnification and centroid proportional to a/m, which is in contrast to the spherically symmetric case where such corrections vanish. The paper then proposes a new system of equations involving lensing observables for the two primary images that allow an observational determination of a/m using gravitational lensing. The resolution capabilities needed to observe this for the Galactic black hole should in principle be accessible to current and near-future instrumentation. This shows that if m is known, which is the case for quite a number of black hole candidates, then the spin a can be determined.

In addition to a black hole's spin, the work also yields a way to determine whether the spinning black hole is actually a naked singularity. In fact, since a black hole becomes a naked singularity for spin-mass ratio of $a/m > 1$, a lensing measurement of a/m gives an observational test of the Cosmic Censorship conjecture for realistic black holes. The technique used in [16] to derive the image properties is based on the degeneracy of the Kerr lens and a suitably displaced Schwarzschild lens at post-Newtonian order. A simple physical explanation for this degeneracy is also given in [16].

* Schwarzschild Black Holes and the Strong-Deflection Bending Angle

Paper [17] extends some of the work in [21, 20] to *strong-deflection* deep inside the potential of a Schwarzschild black hole. One of the fundamental quantities in gravitational lensing is the bending angle of light. Papers [21, 20] give an invariant series for the weak-deflection bending angle of light, where the first order term was the one Einstein found in 1915. These results were extended in paper [17] to the more difficult situation of strong-deflection lensing, where the light ray is close to the black hole.

For Schwarzschild black holes, a perturbation theory is developed in [17] to compute the bending angle going outwards from the photon sphere, i.e., extending beyond the logarithmic deflection term, to infinity. In the process, however, we discovered the surprising result that the standard logarithmic deflection term commonly used in the literature since 1959 was not the most optimal. We gave a new logarithmic deflection term that is more accurate.

Our perturbation framework is also used to reformulate the weak-deflection bending angle series of [21, 20] in terms of a more natural invariant perturbation parameter, one that smoothly transitions between the weak- and strong-deflection bending series.

The paper [17] then presents in invariant form a comparison of the new strong-deflection bending angle series with the numerically integrated exact formal bending angle expression, and finds less than 1% discrepancy for light rays as far out as twice the critical impact parameter. The paper concludes by showing that the strong and weak deflection bending angle series together provide an approximation that is within 1% of the exact bending angle value for light rays traversing anywhere between the photon sphere and infinity.

- **PPN Gravity Models**

Paper [20] develops a formalism for testing Post-Post-Newtonian (PPN) gravitational models by characterizing their gravitational lensing signatures. There are infinitely many such gravity models. The associated metrics are static, spherically symmetric and can be written as a series involving the gravitational radius of the compact object doing the lensing. Working invariantly, we computed corrections to standard weak-deflection lensing observables at first and second order in the ratio of the angular gravitational radius to the angular Einstein ring radius of the lens. We showed that the first-order corrections to the total magnification and centroid position vanish universally for gravity theories that can be written in the PPN framework.

The previous result arises from some surprising, universal relations that we discovered among the lensing observables in PPN gravity models [20]. These universal relations are in terms of the image positions, magnifications, and time delays. A deep consequence is that any violation of the universal relations would signal the need for a gravity model outside the PPN framework. In practical terms, the universal relations can guide observational programs to test general relativity, modified gravity theories, and possibly the Cosmic Censorship conjecture.

We used the new universal relations to identify lensing observables that are accessible to current or near-future technology and to find combinations of observables that are most useful for probing the spacetime metric. We give several explicit predictive applications to the Galactic black hole and the binary pulsar J0737-3039, including a means of knowing observationally when one is probing beyond the weak-deflection limit in a PPN gravity model.

- **Testing for a Possible Fifth Dimension**

  Hyperspace models of gravity postulate that physical space has more than the familiar three dimensions. A relatively simple and concrete example is the type II Randall-Sundrum braneworld model, which will be called *braneworld gravity*. It is a descendant of string theory and postulates that spacetime is *five dimensional* with one dimension of time and four dimensions of space, where the extra spatial dimension is the *fifth dimension*. One of the major challenges faced by such hyperspace theories of gravity is that they are extremely difficult to test. This is a pressing issue, which was eloquently summarized by the string theorist Brian Greene:[4]

  > "... it's hard for me to imagine a discovery that would be more exciting than finding evidence for dimensions [of space] beyond the three with which we're familiar. To my mind, there is currently no other serious proposal whose confirmation would so thoroughly shake the foundation of physics and so thoroughly establish that we must be willing to question basic, seemingly self-evident, elements of reality."

  In paper [19], Keeton and I discovered a new way to test braneworld gravity and, hence, whether physical space has an extra dimension. We had to develop a wave-optics theory of braneworld gravitational lensing. Our theory characterized the interference patterns due to lensing by a tiny primordial braneworld black hole. We gave explicit equations for the locations of the peaks and troughs in the energy spectrum of the interference pattern. In fact, the equations show explicitly how effects from the fourth dimension of space arise in the energy spectrum: contributions from an extra dimension of space would *shift* the interference fringes to lower energies and *reduce* the peak-to-trough separation in the fringe pattern, relative to what would be expected from general relativity. Furthermore, the theoretical framework and testing mechanism we presented allow for generalizations to other hyperspace gravitational models.

  Here is a summary of our test and other findings:

  * **Predicted Signal of Tiny Primordial Braneworld Black Holes**

    We showed that the lensing wave signal due to a primordial braneworld black hole is pronounced when its mass M is of order that of an asteroid or less—say, $M \lesssim 10^{-18} M_\odot$, which has a Schwarzschild radius of order the size of an atomic nucleus. We predicted that if these tiny primordial braneworld black holes exist, then such black holes can gravitationally lens light waves and produce interference fringes in the energy spectra of gamma-ray bursts at energies around $100 \times (M/10^{-18} M_\odot)^{-1}$ MeV. For primordial braneworld black holes with asteroid-like masses, observations at these energies should be accessible to the Fermi Gamma-Ray Space Telescope (formerly called the GLAST satellite). We predicted the explicit "wiggle" in the gamma ray spectrum that the Fermi space telescope satellite should observe. In a cosmological setting, one may also wonder whether the probability of black hole lensing of gamma ray bursts is nontrivial. We pointed out that if primordial braneworld black holes contribute a fraction $\Omega_\bullet$ to the total mass-energy of the cosmos, then the probability that gamma ray bursts are lensed by these objects is roughly $0.1\,\Omega_\bullet$. Current observations do not rule out fractions as high as $\Omega_\bullet \sim 0.1$ [19].

[4] B. Greene, *The Fabric of the Cosmos*, (New York: Vintage Books, 2004), p. 426.

If an interference fringe pattern due to a primordial black hole is observed, then our formulas show that the fringe energies and spacing would yield a simple upper limit on the black hole's mass M. For instance, detection of a black hole that began with primordial mass $M \lesssim 10^{-19} M_\odot$ would challenge general relativity. This is because according to general relativity, primordial black holes in the previous mass range would have evaporated by the present epoch. However, standard braneworld cosmology predicts that primordial braneworld black holes starting with this upper mass limit, and even significantly smaller, could survive to the present time. Such a detection would favor a five-dimensional braneworld model or perhaps some string-theoretic-type hyperspace model of gravity.

* **Nearby Primordial Braneworld Black Holes?**

We also established in [19] that if primordial braneworld black holes exist, have mass M, and contribute a fraction f of the dark matter, then roughly $3 \times 10^5 \times f \times (M/10^{-18} M_\odot)^{-1}$ of them should lie within our Solar System. For example, if 1% of the dark matter in the neighborhood of our Solar System consists of primordial braneworld black holes with mass near $M \sim 10^{-18}\, M_\odot$ (mass of some asteroids), then we predict that there would be roughly $300,000$ such objects in our solar neighborhood. Though these black holes would be harder to detect the closer they are to Earth, any absence of such objects in observations would put constraints on their possible contribution to the distribution of dark matter.

Research Period: 2008–Present

The research program of this section covers stochastic lensing, universal magnification invariants, and Kerr black hole lensing.

Developing a Mathematical Theory of Stochastic Lensing

The previous work dealt largely with the non-random aspects of gravitational lensing. My current research focuses on developing a mathematical theory of stochastic gravitational lensing. A natural first step is to work out such a theory for stochastic microlensing, which was initiated in the papers [12, 13] and determine invariants that help to advance the theory (papers [14, 15]). Stochastic lensing applies to the study of the distribution of dark matter on galactic scales (e.g., papers [22, 23] and references therein) and provides a specific theoretical framework from which to generalize to a broader mathematical study of more general random maps.

- **A Mathematical Theory of Stochastic Microlensing I. Random Time-Delay Functions and Random Lensing Maps [13]**

  Stochastic microlensing is a central tool in probing dark matter on galactic scales. From first principles, we initiate the development of a mathematical theory of stochastic microlensing. Beginning with the random time delay function and associated lensing map, we determine exact expressions for the mean and variance of these transformations. In addition, we derive the probability density function (p.d.f.) of a random point-mass potential, which forms the constituent of a stochastic microlens potential. We characterize the exact p.d.f. of a normalized random time delay function at the origin, showing that it is a shifted gamma distribution, which also holds at leading order in the limit of a large number of point masses if the normalized time delay function was at a general point of the lens plane. For the large number of point masses limit, we also prove that the asymptotic p.d.f. of the random lensing map under a specified scaling converges to a bivariate normal distribution. We show analytically that the p.d.f. of the random scaled lensing map at leading order depends on the magnitude of the scaled bending angle due purely to point masses as well as demonstrate explicitly how this radial symmetry is broken at the next order. Interestingly, we find at leading order a formula linking the expectation and variance of the normalized random time delay function to the first Betti number of its domain. We also determine an asymptotic p.d.f. for the random bending angle vector and find an integral expression for the probability of a lens plane point being near a fixed point. Lastly, we show explicitly how the results are affected by location in the lens plane. The results of this paper are relevant to the theory of random fields and provide a platform for further generalizations as well as analytical limits for checking astrophysical studies of stochastic microlensing.

- **A Mathematical Theory of Stochastic Microlensing II. Random Images, Shear, and the Kac-Rice Formula [12]**

  Continuing our development of a mathematical theory for stochastic microlensing, we explore the expected number of random lensed images of different types. This expectation requires a study of the random microlensing shear, viewed as a random field. We first compute exact expressions for the expectation and variance of the components of the random shear vector due to point masses. We then characterize up to three orders the asymptotic behavior of the joint probability density function (p.d.f.) of the random shear vector due to point masses in the large number of stars limit. At third order, this p.d.f. depends on the magnitude of the shear vector, position in the lens plane, and the star's mass. As a consequence, the p.d.f.s of the shear components are seen to converge, in the infinite number of stars limit, to shifted Cauchy distributions, yielding that the shear components have heavy tails in that limit. We also present the asymptotic p.d.f. of the shear magnitude in the large number of stars limit. All the results on the random microlensing shear are given for a general point in the lens plane. Second, using the co-area proof of the Kac-Rice formula, we derive a formula for the expected number of positive parity images due to a *general* lensing map. This result is employed to deduce similar general formulas for the expected total number of images and the expected number of saddle images. The formulas are applicable to the case of general random distributions of the lenses and light source positions. We apply these formulas to determine the asymptotic global expected number of minimum microimages in the large number of stars regime, where the stars are uniformly distributed. This global expectation is bounded, while the global expected number of images and the global expected number of saddle images diverge as the order of the number of stars.

Universal Geometric Invariants in Gravitational Lensing

The work explores certain geometric invariants in gravitational lensing that transcend lens model type. Such results hold with probability 1 for random lenses and so are also important consistency checks for stochastic lensing results dealing with random image magnification near generic caustics. The findings have application to the detection of dark substructures in galaxies and clusters of galaxies.

- **A Universal Magnification Theorem for Higher-Order Caustic Singularities [15]**

  We prove in [15] that, independent of the choice of a lens model, the total signed image magnification[5] always sums to zero for a source anywhere in the four-image regions of swallowtail, elliptic umbilic, and hyperbolic umbilic caustics. This is a more global and higher-order analog of the well-known fold and cusp magnification relations, in which the total signed magnification in the two-image region of the fold, and the three-image region of the cusp, are both always zero. As an application, we construct a lensing observable for the hyperbolic umbilic magnification relation and compare it with the corresponding observables for the cusp and fold relations using a singular isothermal ellipsoidal lens. We demonstrate the greater generality of the hyperbolic umbilic magnification relation by showing how it applies to the fold image doublets and cusp image triplets and extends to image configurations that are neither. We show that the results are applicable to the study of substructure on galactic scales using observed quadruple images of lensed quasars. The magnification relations are also proved for generic 1-parameter families of mappings between planes, extending their potential range of applicability beyond lensing.

- **A Universal Magnification Theorem II. Generic Caustics up to Codimension Five [14]**

  This paper is a continuation of [15]. We prove in [14] a theorem about magnification relations for all generic general caustic singularities up to codimension five: folds, cusps, swallowtail, elliptic umbilic, hyperbolic umbilic, butterfly, parabolic umbilic, wigwam, symbolic umbilic, 2nd elliptic umbilic, and 2nd hyperbolic umbilic. Specifically, we prove that for a generic family of general mappings between planes exhibiting any of these singularities, and for a point in the target lying anywhere in the region giving rise to the maximum number of real pre-images (lensed images), the total signed magnification of the pre-images will always sum to zero. The proof is algebraic in nature and makes repeated use of the Euler trace formula. We also prove a general algebraic result about polynomials, which we show yields an interesting corollary about Newton sums that in turn readily implies the Euler trace formula. The wide field imaging surveys slated to be conducted by the Large Synoptic Survey Telescope are expected to find observational evidence for many of these higher-order caustic singularities. Finally, since the results of the paper are for generic general mappings, not just generic lensing

[5] Each image magnification is a geometric invariant since it is the reciprocal of the Gaussian curvature at a critical point of the associated time delay surface.

maps, the findings are expected to be applicable not only to gravitational lensing, but to any system in which these singularities appear.

- **A Universal Magnification Theorem III. Generic Caustics up to Codimension Five [11]**

  In the final paper [11] of this series, we extend our results on magnification invariants to the infinite family of A, D, E caustic singularities. We prove that for families of general mappings between planes exhibiting any caustic singularity of the A, D, E family, and for a point in the target space lying anywhere in the region giving rise to the maximum number of lensed images (real pre-images), the total signed magnification of the lensed images will always sum to zero. The proof is algebraic in nature and relies on the Euler trace formula.

- **Orbifolds, the A, D, E Singularities, and Gravitational Lensing [9]**

  We present in [9] a geometric explanation for the existence of magnification relations for the A, D, E family of caustic singularities, which were established in recent work. In particular, it was shown that for families of general mappings between planes exhibiting any of these caustic singularities, and for any non-caustic target point, the total signed magnification of the corresponding pre-images vanishes. As an application to gravitational lensing, it was also shown that, independent of the choice of a lens model, the total signed magnification vanishes for a light source anywhere in the four-image region close to elliptic and hyperbolic umbilic caustics. This is a more global and higher-order analog of the well-known fold and cusp magnification relations. We now extend each of these mappings to weighted projective space, which is a compact orbifold, and show that magnification relations translate into a statement about the behavior of these extended mappings at infinity. This generalizes multi-dimensional residue techniques developed in previous work, and introduces weighted projective space as a new tool in the theory of caustic singularities and gravitational lensing.

Gravitational Lensing by Kerr Black Holes

Recently a lens equation was developed for Schwarzschild black hole lensing with displacement, when the light ray's tangent lines at the source and observer do not meet on the lens plane. In [8] we found a new generalization of this lens equation with displacement to axisymmetric lenses, which includes Kerr black holes, extending the previous work to a fully three-dimensional setting. Our formalism assumes that the source and observer are in the asymptotically flat region, and does not require a small angle approximation. Our analysis in the companion paper [8] also allowed us to go beyond earlier work and probe deeper into the gravitational field of a Kerr black hole, by providing explicit perturbative analytical formulas showing how each lensing observable is affected by higher-order terms. Our results should be useful in observing general relativistic corrections, and can also be used as a tool in testing Einstein's theory and perhaps also Cosmic Censorship.

- **Lensing by Kerr Black Holes. I. General Lens Equation and Magnification Formula [8]**

  In [8], we develop a unified, analytic framework for gravitational lensing by Kerr black holes. In this first paper we present a new, general lens equation and magnification formula governing lensing by a compact object. Our lens equation assumes that the source and observer are in the asymptotically flat region and does not require a small angle approximation. Furthermore, it takes into account the displacement that occurs when the light ray's tangent lines at the source and observer do not meet on the lens plane. We then explore our lens equation in the case when the compact object is a Kerr black hole. Specifically, we give an explicit expression for the displacement when the observer is in the equatorial plane of the Kerr black hole as well as for the case of spherical symmetry.

- **Lensing by Kerr Black Holes. II. Quasi-Equatorial Lensing Observables [7]**

  Paper [7] develops an analytical theory of quasi-equatorial lensing by Kerr black holes. In this setting we solve perturbatively our general lens equation with displacement given in [8], going beyond weak-deflection Kerr lensing to third order in our expansion parameter ε, which is the ratio of the angular gravitational radius to the angular Einstein radius. We obtain new formulas and results for the bending angle, image positions, image magnifications, total unsigned magnification, and centroid, all to third order in ε and including the displacement. New results on the time delay between images are also given to second order in ε, again including displacement. For all lensing observables we show that the displacement begins to appear only at second order in ε. When there is no spin, we obtain new results on the lensing observables for Schwarzschild lensing with displacement.

1.2 Overview of Books

Published Books

- **Gravitational Lensing and Singularity Theory [2]:** This monograph presents a comprehensive and detailed mathematical theory of weak-deflection gravitational lensing and its physical applications in astronomy and cosmology; see pages 8 and for 9 a full discussion.

In keeping with my philosophy of vertically integrating knowledge, I wrote three problem-solving books [4, 5, 6] aimed at high school and elementary school students. These books have Student and Teacher Editions.

- **Scientific Reasoning [4]:** This book introduces high school students to the scientific method and utilizes problem solving to train them in data representation and interpretation, the analysis of research summaries of experiments, and the assessment of conflicts in scientific viewpoints.
- **Algebra, Geometry, and Trigonometry [5]:** This book gives high school students a solid reinforcement in pre-calculus via problem solving in pre-algebra, elementary algebra, intermediate algebra, coordinate geometry, plane geometry, solid geometry, and trigonometry. Similar to [4], students can use the book to prepare for standardized tests such as the ACT and SAT.
- **PSE Mathematics [6]:** This book provides 8th-Grade students with a comprehensive reinforcement of pre-high school mathematics through problem solving.

Continuing with the vertical integration, the next book is a pedagogical contribution targeting upper level undergraduates and entry-level graduate students.

- **An Introduction to Mathematical Finance with Applications: Understanding and Building Financial Intuition [3]:**

  This textbook is aimed at advanced undergraduates and first-year graduate students who are new to finance. The book dissects mathematical models of financial securities and portfolios by isolating their central assumptions and conceptual building blocks, showing explicitly and clearly how their governing equations and relations are derived, applying the models in relevant financial scenarios, and weighing critically the models' strengths and weaknesses. The topics covered include the time value of money, portfolio theory, capital market theory, portfolio risk measures, binomial-tree models of security prices, stochastic calculus, continuous-time security price modeling, the Black-Scholes-Merton model, and the Merton jump diffusion model. The book keeps a good balance between rigorousness and depth in mathematics on the one hand and its applications in finance on the other.

Book in Preparation

- **Gravitational Lensing and Black Holes [1]:** This monograph will present a mathematical theory of gravitational lensing by black holes. The book covers the following topics: optical geometry and the Schwarzschild black hole, lensing observables for the Schwarzschild black hole, Post-Post Newtonian lensing, optical geometry and the Kerr black hole weak-deflection Kerr lensing, Kerr lensing to higher order lensing by gravitational waves and exotic gravity, and magnification relations. The monograph is a natural continuation of *Singularity Theory and Gravitational Lensing* [2], which dealt exclusively with weak-deflection gravitational lensing.

  This book is under contract with Springer. Estimated completion date: 2018.

1.3 List of Publications

Books

The books are listed in reverse chronological order in each category.

Research Level

[1] **Gravitational Lensing and Black Holes**
M. C. Werner and **A. O. Petters** (Springer, New York, 2018). In preparation.

[2] **Singularity Theory and Gravitational Lensing**
A. O. Petters, H. Levine, and J. Wambsganss (Birkhäuser-Springer, Boston, 2001).
Progress in Mathematical Physics Series (Volume 21).

Advanced Undergraduate/Beginning Graduate Level

[3] **An Introduction to Mathematical Finance with Applications**
A. O. Petters and X. Dong (Springer, SUMAT Series, New York, 2016).

High School School Level

[4] **Scientific Reasoning: Student Edition**
A. O. Petters (BRC, Benque-Belize, 2007)

Scientific Reasoning: Teacher Edition
A. O. Petters (BRC, Benque-Belize, 2007)

[5] **Algebra, Geometry, and Trigonometry: Student Edition**
A. O. Petters (BRC, Benque-Belize, 2007)

Algebra, Geometry, and Trigonometry: Teacher Edition
A. O. Petters (BRC, Benque-Belize, 2007)

Elementary School Level

[6] **PSE Mathematics: Student Edition**
A. O. Petters (BRC, Benque-Belize, 2007)

PSE Mathematics: Teacher-Tutor-Parent Edition
A. O. Petters (BRC, Benque-Belize, 2007)

Papers

The papers are listed in reverse chronological order.

[7] **Lensing by Kerr Black Holes. II. Quasi-Equatorial Lensing Observables**
A. B. Aazami, C. R. Keeton, and **A. O. Petters**, *J. Math. Phys.* **52**, 102501 (2011). Web archive (astro-ph): arXiv: 102.4304.

[8] **Lensing by Kerr Black Holes. I. General Lens Equation and Magnification Formula**
A. B. Aazami, C. R. Keeton, and **A. O. Petters**, *J. Math. Phys.* **52**, 092502 (2011). Web archive (astro-ph): arXiv: 1102.4300.

[9] **Orbifolds, the A, D, E Family of Caustic Singularities, and Gravitational Lensing**
A. B. Aazami, **A. O. Petters**, and J. Rabin, *J. Math. Phys.* **52**, 022501 (2011). Web archive (math-ph): arXiv:1004.0516v1.

[10] **Mathematics of Gravitational Lensing: Multiple Imaging and Magnification**
A. O. Petters and **M. C. Werner**, *Gen. Rel. and Grav.*, Special Issue on Gravitational Lensing (2010). Web archive (astro-ph): arXiv:0912.0490v1.

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A. O. Petters, A. Teguia, and B. Rider, *J. Math. Phys.* **50**, 122501 (2009). Web archive (astro-ph): arxiv.org/abs/0807.4984v2.

[13] **A Mathematical Theory of Stochastic Microlensing. I. Random Time-Delay Functions and Lensing Maps**
A. O. Petters, A. Teguia, and B. Rider, *J. Math. Phys.* **50**, 072503 (2009). Web archive (astro-ph): arxiv.org/abs/0807.0232v2.

[14] **A Universal Magnification Theorem. II. Generic Caustics up to Codimension Five**
A. B. Aazami and **A. O. Petters**, *J. Math. Phys.* **50**, 082501 (2009). Web archive (math-ph): arxiv.org/abs/0904.2236v4.

[15] **A Universal Magnification Theorem for Higher-Order Caustic Singularities**
A. B. Aazami and **A. O. Petters**, *J. Math. Phys.* **50**, 032501 (2009). Web archive (astro-ph): arXiv.org/abs/0811.3447v2.

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M. C. Werner and **A. O. Petters**, *Phys. Rev. D*, **76**, 064024 (2007). Web archive (gr-qc): xxx.lanl.gov/abs/0706.0132.

[17] **Light's Bending Angle due to Black Holes. From the Photon Sphere to Infinity**
S. I. Iyer and **A. O. Petters**, *Gen. Rel. and Grav.*, **39**, 1563 (2007). Web archive (gr-qc): xxx.lanl.gov/abs/gr-qc/0611086.

[18] **Testing Theories of Gravity with Black Hole Lensing**
C. Keeton and **A. O. Petters**, in *Proceedings of the Eleventh Marcel Grossmann Meeting on General Relativity*, ed. R. Ruffini (World Scientific, Singapore, 2006).

[19] **Formalism for Testing Theories of Gravity Using Lensing by Compact Objects. III. Braneworld Gravity**
C. Keeton and **A. O. Petters**, *Phys. Rev. D*, **73**, 104032 (2006). Web archive (gr-qc): xxx.lanl.gov/abs/gr-qc/0603061.

[20] **Formalism for Testing Theories of Gravity Using Lensing by Compact Objects. II. Probing Post-Post-Newtonian Metrics**
C. Keeton and **A. O. Petters**, *Phys. Rev. D*, **73**, 044024 (2006). Web archive (gr-qc): xxx.lanl.gov/abs/gr-qc/0601053.

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C. Keeton and **A. O. Petters**, *Phys. Rev. D*, **72**, 104006 (2005). Web archive (gr-qc): xxx.lanl.gov/abs/gr-qc/0511019.

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C. Keeton, S. Gaudi, and **A. O. Petters**, *Astrophys. J.* **635**, 35 (2005). Web archive (astro-ph): xxx.lanl.gov/abs/astro-ph/0503452.

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A. O. Petters, *Mon. Not. R. Astron. Soc.* **338**, 457 (2003). Web archive (astro-ph): xxx.lanl.gov/abs/astro-ph/0208500.

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B. S. Gaudi and **A. O. Petters**, *Astrophys. J.* **580**, 468 (2002). Web archive (astro-ph): xxx.lanl.gov/abs/astro-ph/0206162.

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H. Levine, **A. O. Petters**, and J. Wambsganss, *J. Math. Phys.* **34**, 4781 (1993).

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A. O. Petters and D. Spergel, in *Gravitational Lenses*, eds. R. Kayser, T. Schramm, and L. Nieser (Lecture Notes in Physics **406**, Springer, Berlin, 1992).

[52] **Morse Theory and Gravitational Microlensing**
A. O. Petters, in *Gravitational Lenses*, eds. R. Kayser, T. Schramm, and L. Nieser (Lecture Notes in Physics **406**, Springer, Berlin, 1992).

1.4 Grants

- **2007–2011: NSF Grant DMS-0707003** (Geometric Analysis)
  Geometric Analysis, Wave Optics, and Geometric Gravity
  Principal Investigator/Project Director: *A. O. Petters*
  Duke University, Department of Mathematics
  Total: \$353,000. Period: 9/1/07–8/31/11.

- **2004–2009: NSF Grant AST-04344277/-0433809** (Astronomy)
  Collaborative Research: The Mathematics of Stochastic Gravitational Lensing—Applications to Flux Ratio Anomalies and Dark Matter
  Principal Investigator/Project Director: *A. O. Petters*
  Duke University, Department of Mathematics
  Total: \$600,000 (Duke \$400,000, MIT \$200,000). Period: 7/1/04–6/30/09.

- **2003–2007: NSF Grant DMS-0302812** (Geometric Analysis)
  Random Gaussian Curvatures, Image Centroids, and Caustic Surfaces in Gravitational Lensing
  Principal Investigator/Project Director: *A. O. Petters*
  Duke University, Department of Mathematics
  Total: \$135,000. Period: 7/1/03–6/30/07.

- **1998–2003: NSF CAREER Grant DMS-9896274** (Geometric Analysis)
  Gravitational Lensing Geometry and Optics
  Principal Investigator/Project Director: *A. O. Petters*
  Duke University, Department of Mathematics
  Total: \$211,254. Period: 7/1/98–6/30/03.

- **1998–2002: Alfred P. Sloan Research Fellowship** (# BR-3772)
  Mathematical Theory of Gravitational Lensing
  Principal Investigator/Project Director: *A. O. Petters*
  Duke University, Department of Mathematics
  Total: \$35,000. Period: 9/16/98–9/15/02.

- **1994–1997: NSF Grant DMS-9404522** (Geometric Analysis)
  Singularity Theory and Gravitational Lensing
  Principal Investigator/Project Director: *A. O. Petters*
  Princeton University, Department of Mathematics
  Total: \$49,998. Period: 7/15/94–12/31/97.

1.5 Talks

This list is in reverse chronological order and includes only talks with properly recorded dates.

- Duquesne University, Physics Department, Einstein Centennial Lecture (Pittsburg, September 24, 2015)
- National Math Festival by the Mathematical Sciences Research Institute and the Institute for Advanced Study (Washington DC—Smithsonian, April 18, 2015)
- University of Arkansas, Mathematics Department Spring Lecture Series (Fayetteville, April 4, 2013)
- TEDxNCSSM Lecture, Is There a Fifth Dimension? (Durham, May 3, 2012)
- Science Lecture, Duplin Early College High School (Duplin, February 25, 2011)
- University of South Florida, Mathematics Department Seminar (Tampa, Florida, April 2, 2010)
- University of South Florida, Nagle Lecture (Tampa, Florida, April 1, 2010)
- Tapia 70 Conference (Rice University, Houston, Texas, May 29, 2009)
- Penn State University—College Station, Penn State Forum Lecture Series (College Station, Pennsylvania, March 11, 2009)
- North Park University, Arts and Sciences Campus Theme Lecture Series (Chicago, Illinois, October 2, 2008)
- Johnson C. Smith University, Lyceum Lecture Series (Charlotte, North Carolina, September 24, 2008)
- City University of New York—Hunter College, Science 200 Lecture Series (New York, New York, September 10, 2008)
- Francis Marion University, Department of Mathematics Conference (Florence, South Carolina, March 30, 2007)
- NASA Goddard Space Center and University of Maryland—College Park, 2007 Eyes on the Sky Colloquium Series (College Park, Maryland, March 8, 2007)
- University of North Carolina—Chapel Hill, Twelfth Conference for African-American Researchers in the Mathematical Sciences (Chapel Hill, North Carolina, June 20, 2006)
- Institute of Mathematics and its Applications, University of Minnesota (Minneapolis, Minnesota, March 22, 2006)
- Duke University, Einstein Week Lecture, Celebration of the 100th Birthday of Relativity Theory (Durham, North Carolina, September 6, 2005)
- University of Texas—Arlington, MAA Invited Address, 85th Meeting of the Texas Section of the MAA (Arlington, Texas, April 15, 2005)
- Arizona State University—Tempe, Mathematics Awareness Month, Mathematics and the Cosmos, Department of Mathematics (Tempe, Arizona, April 13, 2005)
- Emory University, Southeastern Geometry Seminar, Department of Mathematics (Atlanta, Georgia, March 16, 2005)
- Centre International de Rencontres Mathématiques, Workshop on Applications of Singularity Theory (Luminy-Marseille, France, February 11, 2005)

- University of Michigan, Mathematics Colloquium (Ann Arbor, Michigan, April 20, 2004)
- University of Miami, Math-Physics Colloquium (Miami, Florida, April 8, 2004)
- Purdue University, Math-Physics Colloquium (Lafayette, Indiana, March 30, 2004)
- Duke University, Physics Colloquium (Durham, North Carolina, March 3, 2004)
- University of Massachusetts—Lowell, Physics Colloquium (Lowell, Massachusetts, October 29, 2003)
- SACNAS Conference, Keynote Address (Albuquerque, New Mexico, October 4, 2003)
- MIT, Physics Seminar (Cambridge, Massachusetts, May 1, 2003)
- Rice University, President's Lecture Series by Diverse Scholars (Houston, Texas, March 21, 2003)
- Harvard University, Mather House Lecture Series (Cambridge, Massachusetts, February 20, 2003)
- Spelman College, NSBP Conference (Atlanta, Georgia, February 14, 2003)
- MSRI, Blackwell-Tapia Prize Lecture (Berkeley, California, November 2, 2002)
- Duke University, Nuclear/Particle Theory Seminar (Durham, North Carolina, May 1, 2002)
- Cornell University, TAM-IGERT Colloquium (Ithaca, New York, February 16, 2001)
- UCLA, NAM/MAA David Blackwell Invited Address (Los Angeles, California, August 5, 2000)
- University of Texas—Dallas, Math-Physics Seminar (Dallas, Texas, June 30, 2000)
- Lensing Conference, Recent Progress and Future Goals (Boston, Massachusetts, July 26, 1999)
- Stanford University, Math-Physics Colloquium (Palo Alto, California, April 6, 1998)
- Brown University, Math-Applied Math-Physics Colloquium (Providence, Rhode Island, March 16, 1998)
- Columbia University, Math Colloquium (New York, New York, February 24, 1998)
- Duke University, Math-Physics Colloquium (Durham, North Carolina, January 9, 1998)
- Cairo University, Math-Physics Colloquium (Cairo, Egypt, June 30, 1997)
- Marcel Grossmann Meeting on General Relativity (Jerusalem, Israel, June 23, 1997)
- Virginia State University, Math-Physics Colloquium, Guest Speaker for Black-History Month (Petersburg, Virginia, February 11, 1997)
- Fourth Invitational Mathematics Meeting, National Security Agency (Fort G. Meade, Maryland, November 17, 1996)
- Second World Congress of Nonlinear Analysts (Athens, Greece, July 10, 1996)
- Princeton University, Math Colloquium (Princeton, New Jersey, November 29, 1995)
- Seventh Annual Symposium on Frontiers of Science, National Academy of Sciences (Irvine, California, November 2, 1995)
- University of Bristol, Michael Berry's Research Group, H. H. Wills Physics Laboratory (Bristol, United Kingdom, August 29, 1995)
- International Conference on General Relativity and Gravitation (Florence, Italy, August 6, 1995)
- Oxford University, Roger Penrose's Relativity Group, Mathematical Institute (Oxford, United Kingdom, August 18, 1995)

- First Conference for African-American Researchers in the Mathematical Sciences, M.S.R.I. (Berkeley, California, June 21, 1995)
- Caltech, Theoretical Astrophysics Seminar, Roger Blandford's Research Group (Pasadena, California, April 20, 1995)
- Observatoire de Paris-Meudon, Astrophysics Seminar (Meudon, France, March 21, 1995)
- University of Minnesota—Minneapolis, Math Seminar, The Geometry Center (Minneapolis, Minnesota, November 3, 1994)
- Max-Planck Institut für Astrophysik, Astrophysics Seminar (Garching, Germany, June 15, 1994)
- Brandeis University, Math Conference in Honor of Harold Levine's Retirement (Waltham, Massachusetts, April 29, 1994)
- Princeton University, Mathematics Seminar (Princeton, New Jersey, March 1, 1994)
- Bell Labs - Lucent Technologies, Math Seminar, (Murray Hill, New Jersey, February 21, 1994)
- Brown University, Joint L.C.D.S. and Math/Physics Seminar (Providence, Rhode Island, October 26, 1992)
- Northeastern University, Math Colloquium (Boston, Massachusetts, May 22, 1992)
- Brandeis University, Special Math Lecture (Waltham, Massachusetts, April 16, 1992)
- Howard University, Math Colloquium (Washington, D.C., April 10, 1992)
- University of Massachusetts—Boston, Ronald E. McNair Program Seminar Series (Boston, Massachusetts, February 22, 1992.
- MIT, Mathematics Seminar (Cambridge, Massachusetts, 1991)
- York College, CUNY, Physics Colloquium (Queens, New York, 1990)
- Bell Labs, Lucent Technologies, Math Colloquium (Murray Hill, New Jersey, 1988)
- NIH Conference (Washington, D.C., 1986)

1.6 Conferences, Workshops, and Sessions Organized

- Race in Space, Session Chair, Duke University (Durham, October 25-26, 2013)
- Dark Matter, Complex Methods, and Orbifolds in Gravitational Lensing Workshop, Co-Organizer, Petters Research Institute (Dangriga, Belize, March 19–23, 2010)
- Probability and its Lensing Applications, Workshop Co-Organizer, Petters Research Institute (Dangriga, Belize, December 16–22, 2008)
- Magnification Profile of Elliptic Umbilics, Workshop Organizer, Petters Research Institute (Dangriga, Belize, August 4–8, 2008)
- Singularities and Magnification Cross Section, Workshop Organizer, Petters Research Institute (Dangriga, Belize, March 7–11, 2008)
- Geometric and Stochastic Lensing, Workshop Co-Organizer, Petters Research Institute (Dangriga, Belize, January 5–12, 2007)
- Blackwell-Tapia 2006 Conference, Conference Co-Organizer, Institute of Mathematics and its Applications, University of Minnesota (Minneapolis, Minnesota, November 3–4, 2006)
- Einstein Week, Centennial Celebration of Relativity Theory, Conference Co-Organizer, Departments of Mathematics and Physics, Duke University (Durham, North Carolina, September 5–10, 2005)
- Tenth Conference for African-American Researchers in the Mathematical Sciences, Session Chair and Organizer, MSRI and Lawrence Livermore Laboratories (Berkeley, California, June 22–25, 2004)
- Seventh Conference for African-American Researchers in the Mathematical Sciences, Conference co-Organizer, Duke University (Durham, North Carolina, June 22–25, 2001)
- Gravitational-Lenses, Session Chair and Organizer, The Ninth Marcel Grossmann Meeting on General Relativity, La Sapienza (Rome, Italy, July 4, 2000)
- Gravitational-Lenses, Session Chair and Organizer, The Eighth Marcel Grossmann Meeting on General Relativity Hebrew University (Jerusalem, Israel, June 23, 1997)
- Gravitational-Lenses, Session Chair and Organizer, The Seventh Marcel Grossmann Meeting on General Relativity Stanford University (Stanford, California, California, July 24, 1994)

2 Teaching

2.1 Teaching Statement

Philosophy, Goals, and Approaches

My teaching philosophy is that professors are *facilitators* of the learning process, who often play the role of coach. I have a lot of regard for my students and encourage them to contribute actively to the learning process, especially through their fresh questions and unbiased outlook.

Two broad goals I have in mathematics teaching are to help students **discover** how to do the following:

- **Think precisely, logically, and abstractly; compute efficiently** (even with structures more complicated than numbers); **and communicate mathematical reasoning effectively.** In the process, I also try to foster an appreciation for the roles of different mathematical modes of thinking (analytical, algebraic, probabilistic, etc.) and cultivate an intuition for the beauty with which mathematics fits together and flows.
- **Apply mathematics to solve problems in a variety of fields.** I strive to have students obtain an appreciation for the power of mathematical abstraction by having them see how the exact mathematical technique or theorem can be used in fields as diverse as ecology, computer science, finance, astrophysics, etc. I also aim to have them develop an ability to separate out the extraneous elements from a complex real-world problem and focus rigorously on the underlying core mathematical issue.

The approaches that have worked for me in achieving the above goals are as follows:

- **Know students' background.** On the first day of class, I have all my students introduce themselves and talk about the math courses they have taken, why they are taking my class, and their academic goals at Duke. This helps me tailor the flow of the lectures to fit the class's math profile.
- **Employ a bottom-up approach.** I start from concrete examples and work my way systematically to general theorems and techniques. Whenever possible, my choice of examples adjusts to reflect the academic and professional interests of the students.
- **Have students work through a lot of problems.** This is the bread-and-butter of mathematics teaching. Along with mastering important mathematical methods and ideas, problem-solving allows students to see different techniques repeated in a variety of situations and appreciate how and why these techniques were webbed together into a general theory. Problem-solving also enables students to learn how to employ mathematical reasoning to guide sophisticated symbolic and numerical software in addressing complicated problems that are intractable without computer technology.
- **Encourage working in groups.** This is a critical skill in today. It also helps to alleviate some students' the anxiety about being able to get through the material successfully. For example, one can have students work together on certain homework sets that are not collected, but give quizzes that come directly from them. This teaches students about responsibility for their own learning outside the classroom and helps them discover how to negotiate the currents of group dynamics.

- **Pay attention to the mechanics of teaching.** I go to great pains to present well organized lectures, write legibly on the blackboard (especially given the many exotic symbols in mathematics), and make sure that what I am saying is in step with what I am writing. Students quickly lose confidence in professors whose lectures are poorly organized and have frequent mistakes.

In addition to the above approaches, I am available to students, clear about my expectations, and meticulous in making sure that I assign awards consistent with these expectations.

Introduced Interdisciplinary Area

In 1998, I introduced gravitational lensing as a new interdisciplinary research area at Duke, one that bridges mathematics and astronomy. The field is also quite intra-disciplinary, drawing on several areas within mathematics—differential geometry, singularity theory, complex analysis, probability theory, etc. Since Duke has one of the best undergraduate mathematics programs in the nation, my approach was to start with our mathematics undergraduates.

To target the undergraduate mathematics audience, I created the *Mathematics of Gravitational Lensing* seminar cited above. It took effort to encourage students to enroll in the seminar since the majority of the math majors at that time were not engaged in physics related work; most were focused on only a specific area in mathematics or mathematics linked to economics and computer science. The students enjoyed the seminar as they saw the many links within and outside mathematics that the subject offers. They in turn spread the work about the field to their friends. Since then *ten* Duke undergraduates have had *long-term* independent research projects with me in gravitational lensing. These were not semester research projects; each was quite involved and lasted for no less than a year. The students voluntarily extended their independent study each semester. In a similar way, my new *General Relativity* graduate course has attracted graduate students to gravitational lensing. This effort has culminated with the field becoming an active research area at Duke, where my Research Group has had collaborations with researchers at Cambridge University, MIT, Potsdam University, Rutgers University, the State University of New York at Geneseo, and the University of Colorado at Boulder.

I have supervised 22 undergraduate research projects (see page 46). The students were also mentored and have pursued a diverse array of careers. Here are some examples:

- **Wifag Adnan** earned a B.A. from Duke. She went on to Princeton University to receive a Ph.D. in an economics.
- **Peter Blair** won a Goldwater Scholarship, APS Fellowship, and Bell Laboratories Fellowship. He earned a B.S. from Duke and is currently pursuing a Ph.D. in applied economics at The Wharton School, University of Pennsylvania.
- **Gil Libling** earned a B.A. and M.B.A. from Duke. He has worked at several investment banks, including Lehman Brothers and Morgan Stanley.
- **Samuel Malone** won a Rhodes Scholarship, Goldwater Scholarship, and Duke Faculty Scholar Award. He earned a B.S. from Duke and Ph.D. in economics from the University of Oxford.

- **Michelle Sowemimo** earned a B.A. from Duke and J.D. from the University of Chicago.
- **Luke Stewart** earned a B.S. from Duke and M.B.A. from Stanford University. He is currently working for Cisco.
- **Jay Strader** earned a B.S. from Duke and Ph.D. in astronomy from the University of California, Santa Cruz. He went on to become a Hubble Fellow at the Harvard-Smithsonian Center for Astrophysics.

Synergies Between Teaching and Research

I always seek to create synergies between my teaching and research. Indeed, the impact of research on teaching is well known—e.g., introducing new courses, updating textbooks to reflect cutting edge results, etc. Conversely, graduate level teaching can further research through seminars that deal with open questions in one's field. Although the influence of undergraduate teaching on mathematical research is less obvious, I have found undergraduate teaching to have a subtle impact on my research. For example, freshman-sophomore undergraduate courses like calculus and linear algebra are overviews of some of the most successful mathematical theories ever developed and applied to the sciences. Knowing the detailed architecture of these theories gave me a lot of insight into how to assemble effectively a mathematical theory of gravitational lensing (see *Research Overview*, page 5). Undergraduate teaching also helps to clarify core concepts and keeps the fundamental theorems fresh in one's mind, which are important to my research, since I often need to invoke results and techniques from many different areas of mathematics. Overall, the synergy between teaching and research has continued to fuel me as a scholar-teacher.

2.2 New Courses Developed

I developed the following new courses at Duke, except the last one which was created at Princeton:

[1] **Financial Derivatives** (Math 582/Econ 674; created fall 2013; advanced undergraduate and first-year graduate)

This rigorous introduction to financial derivatives starts with the modeling of security prices using binomial trees and geometric Brownian motion. European Options, American options, forwards, and futures are then treated along with put-call parity. It is followed by an introduction to the Black-Scholes-Merton (BSM) Model, which includes Delta and Gamma hedging and the volatility index VIX. The BSM pricing formula is then derived by a binomial-tree, risk-neutral-expectation, and p.d.e. approach. A critique is given of the BSM model and extensions to underliers with discontinuous prices and stochastic volatility are treated using the Merton jump diffusion, Heston, and GARCH models.

[2] **Topics in Mathematical Finance** (Math 690-42; created fall 2013; advanced undergraduate and first-year graduate)

This course gives students an opportunity to probe deeper, in teams or individually, topics in quantitative finance of interest to them. It can serve as a means to initiate a research project, work through a paper in progress, or even learn about a specific model(s) one needs for later research. Topics can deal with financial modeling issues drawn from academic foundational problems (e.g., stochastic volatility, security price modeling, incomplete markets) as well as applied problems from governmental and state agencies (e.g., management of state pension funds), the nonprofit sector (e.g., the World Bank), foreign markets (e.g., securities markets in BRIC nations, modeling financial products and portfolio management in a non-Western context), non-proprietary problems from the private sector, etc. The format of the course's dynamics will be tailored to the registered students. The course culminates with each team or individual (if one elects not be in a team) producing a paper, which will be presented to the class.

[3] **Quantitative Finance: The Black-Scholes-Model and Beyond** (Finance 491.301; created spring 2009; MBA)

I developed this course at the Fuqua School of Business. It explores in depth the most generalized Black-Scholes-Merton option pricing model and its extensions to stochastic interest rates and stochastic volatility.

[4] **General Relativity** (Math 236/Physics 292; created fall 2002; graduate)

I co-developed this course as a joint offering between the mathematics and physics departments. It treats the fundamentals of general relativity, covering Einstein's equations and the curvature of spacetime with applications to cosmology and black holes. This course provides a rich arena for students to develop their mathematical and physical intuition, and to experience how both insights can come to bear significantly on important physical problems.

[5] **Mathematical Finance with Applications** (Math 215; developed in fall 2001; advanced undergraduate and first-year graduate)

I developed this course in the mathematics department. This course is broad introduction to mathematical finance, covering the time value of money, portfolio theory, capital market theory, the modeling of security prices, and the Black-Scholes-Merton model. It attracted students from mathematics, economics, computer science, engineering, physics, and Fuqua Business School.

[6] **Mathematics of Gravitational Lensing** (Math 196S; created fall 1999; advanced undergraduate)

The course deals with applications of Morse theory to image counting problems in gravitational lensing. It had students from mathematics, physics, and computer science.

[7] **Mathematics of Light Deflection** (Math 331; created fall 1995; advanced undergraduate)

I developed this course at Princeton. The course explores the mathematical framework for how gravity acts on light at it propagates across the universe. It had students from mathematics and physics.

2.3 List of Courses Taught

Duke University (1998–2002, 2004–present)

Trinity College of Arts and Sciences

- **Developed New Course:**
  Financial Derivatives (Math 582/Econ 674; taught spring 2014; advanced undergraduate and first-year graduate)
- **Developed New Course:**
  Mathematical Finance (Math 581/Econ 673, Math 215/Econ 225; taught each fall of 2001, 2004, and 2006; spring 2008; falls of 2008-2011, 2013; advanced undergraduate and first-year graduate)
- **Developed New Graduate Course:**
  Mathematical Finance (Math 581/Econ 673, Math 215/Econ 225; taught each fall of 2001, 2004, and 2006; spring 2008; falls of 2008-2011, 2013; advanced undergraduate and first-year graduate)
- Freshman Seminar: Finance (Math 89S; spring 2013)
- Topics in Probability: The Black-Scholes-Merton Model and Beyond (Math 690, Math 288; summers 2012, 2013; advanced undergraduate and first-year graduate)
- Probability Theory (Math 135/Stat 104; spring 2005, spring 2008; undergraduate)
- Linear Algebra and Differential Equations (Math 107; spring 2006; undergraduate)
- General Relativity Seminar (Physics 222S; fall 2004; undergraduate)
- **Co-Developed New Course:**
  General Relativity (Math 236/Physics 292; fall 2002; graduate level)

- Linear Algebra and Applications (Math 104; spring 1999, every fall of 2000–2002; undergraduate)
- Advanced Calculus I (Math 139; fall 2000; undergraduate)
- **Developed New Undergraduate Seminar:**
  Mathematics of Gravitational Lensing (Math 196S; fall 1999; advanced undergraduate)
- Multivariable Calculus (Math 103; fall 1998; undergraduate)

Fuqua School of Business

- Finance Concentration Project (Finance 897E-001, Finance 480E.001, Executive MBA program, 2010-2014)
- **Developed New Course:**
  Quantitative Finance: Special Topics (Finance 491.301, Daytime MBA, Term 3, 2009, 2010)
- Global Consulting Practicum (Strategy 490.302, Daytime MBA, Term 3, 2010)

Princeton University (1993–1998, 2007)

- Regression and Applied Time Series (ORFE 405; fall 2007).
- Advanced Linear Algebra with Applications (Math 204; fall 1995–fall 1997; undergraduate)
- **Developed New Undergraduate Course:**
  Mathematics of Light Deflection in the Universe (Math 331; fall 1995 and 1996; advanced undergraduate)
- Multivariable Calculus (Math 201; fall 1994).
- Calculus II (Math 104; spring 1994; undergraduate)
- Calculus I (Math 103; fall 1993; undergraduate)

MIT (1991–1993, 2003–2004)

- Relativity (Physics 8.033; fall 2003).
- General Relativity (Physics 8.962; spring 2003; graduate)
- Advanced Calculus for Engineers II (Math 18.076; spring 1993; advanced undergraduate and first-year graduate)
- Advanced Calculus for Engineers I (Math 18.075; spring, fall 1992; advanced undergraduate and first-year graduate)
- Complex Variables with Applications (Math 18.04; fall 1992; undergraduate)
- Multivariable Calculus (Math 18.02; fall 1991, spring 1992; undergraduate)

3 Administrative and Service Experience

3.1 Administrative and Service Overview

Over the past 20 years, I have gained in-the-trenches experiences covering numerous aspects of the academic administrative enterprise at top-ten, private, research universities in the U.S. Some examples:[6]

- hiring, promotion, and tenure of faculty
- chairing a committee evaluating faculty for endowed chairs in the humanities, social sciences, and natural sciences
- hiring and reviewing senior administrators
- servicing on budgetary committees
- directing an undergraduate degree program (Director of Undergraduate Studies)
- directing a graduate degree program (Director of Graduate Studies)
- directing an undergraduate scholarship program
- serving as pre-major adviser to undergraduates
- serving as Faculty-In-Residence in student dormitories
- recruiting and mentoring underrepresented minority students and faculty
- making presentations to fundraisers
- serving as president of a non-profit research institute in Belize
- chairing the Belize Prime Minister's Council of Science Advisers

The remainder of this section discusses some unique aspects of my service contribution to Duke's mathematics department, student residential life, and the underrepresented minority community.

Sample of Service to Department

First, I recruited underrepresented minority students **Andrea Watkins** and **Alberto Teguia** to Duke's Mathematics graduate program. This recruitment is significant because historically *Andrea and Alberto were the second and third black students, respectively, to graduate from Duke's Mathematics Ph.D. program.* I served as a mentor to both students with Alberto being one of my Ph.D. students. Andrea graduated in 2010 and Alberto in 2011.

Second, shortly after arriving at Duke in the fall of 1998, I saw a need for *mathematical finance* as an interdisciplinary course offering in our department. This led me to organize a Duke/Goldman

[6] A detailed list of the administrative and service activities is in Section 3.2, while the international service work is in Section 4.

Sachs conference in early spring 1999 with the support of the Duke Administration and our Departmental Chair at the time. I brought in a team of mathematical finance experts from Goldman Sachs, who made presentations to our undergraduates, graduate students, postdoctoral fellows, and faculty about various facets of the financial services industry. In particular, they advised us on the latest mathematical tools and algorithms relevant to investment banking and securities firm research, and even recruited some of our mathematics students and postdoctoral fellows. Roughly 100 people from across the university attended.

Following the Duke/Goldman Sachs conference, I continued a good relationship with the firm's Steve Duncker (Managing Director) and Donna Winston (Vice President). *This resulted in Goldman Sachs giving at least ten*[7] *scholarships to underrepresented-minority Duke undergraduates.* I was delighted when in 2001, they wrote to me saying, "You have been an amazing resource to us and we would like to show our appreciation by supporting an area of the University that is important to you." After consultation with my departmental chairman and the Development Office, *I facilitated a $25,000 donation from Goldman Sachs to my department.* This donation assisted with supporting the teaching of the new course in fall of 2001 on mathematical finance that I developed after the conference.

Sample of Service to University Life

I would like to describe in particular my service to Duke's residential life as **Faculty-in-Residence** (1999-2003, 2004-2006) in Wilson Hall and Bassett Hall. That role was special because it allowed me to be embedded in student life by living in an apartment in a student dormitory. I was a mentor who served as a bridge between them and the faculty, and facilitator of the intellectual climate in the dormitory.

Below are some examples of the variety of student events I organized and hosted:

- **visits by two Nobel Prize winners in physics** — Val Fitch and Russell Hulse — and **two-time Pulitzer Prize winning journalist** Jon Franklin. These events were attended not only by Wilson students, but also by students and faculty from across the campus. At these events the speakers not only spoke about the content of their work, but also described the personal struggles they had with the research problems and issues that eventually brought them such high honors. One speaker even gave us an insider's view of the whole process from arrival in Stockholm to the Nobel Prize ceremony.

- **visits by groups of investment bankers**, who included several Duke alumni. They met with our students telling them about the pros and cons of life on Wall Street, and how best to prepare for such a career. These events provided special networking opportunities for our students.

- **discussion on the academic life of professors**, where the students learned about the journey to tenure, how professors try to balance research and teaching, including what professors expect from their students. The students find this fascinating and get an even better appreciation for the pressures professors are under.

- **visit by a group of graduate students**, who spoke about their academic life, including why they went to graduate school, how it compares with being an undergraduate, how they

[7] The total number is actually more. A partial list covering 2000-2003 is on page 43.

prepared for graduate school, the pros and cons of taking a job before going on to graduate school, etc. Similar to the "Life of a Professor" event, the students really appreciate getting a glimpse into life of graduate students, who are often their TAs for courses.

- **visit by a group of sophomores, juniors, and seniors**, who gave words of advice ranging from the transition to living on West Campus to how to select courses and "good" professors.
- **visit by the players on the men and women's basketball teams.** Our students were especially delighted not only to meet the players, but also to have a Q&A period where the players spoke about the challenges of balancing sports and academics.

Most of the above events were held in our Wilson and Bassett Hall apartments. During finals, we would have evening study breaks in our apartment, providing high quality coffee, tea, and desserts, which the students really appreciate during such a time of anxiety and stress.

In the dorms, I also served as mentor and supervisor of undegraduates who wanted a research experience. For example, in fall 2001, I supervised a highly exceptional first-year undergraduate, **Vanessa Rousso**. Through a Dananberg scholarship, she did a research project with me on the *Structure and Function of Electronic Exchange Networks for Financial Markets.* Interestingly, since graduating from Duke, Vanessa has become a worldclass poker player. Wikipedia cites, "At the age of 27, she ranks among the top five women in poker history in terms of all-time money winnings."

As Faculty-in-Residence, I was mindful to provide a good balance between educational and fun events. For example, we hosted dinners (some with international dishes) and invited the students over numerous times to watch the Duke basketball games. With the educational events, we were also careful to not have them seem like just another seminar/colloquium, so we encouraged our guests to talk both about their work and the personal aspects of being a practitioner in that field. Overall, I was quite happy living in the residential halls. I especially took great pleasure in observing how our first-year students transformed quickly from feeling like timid newcomers to comfortably engaging the academic and student life possibilities at Duke.

Sample of Service to the Underrepresented Minority Community

My service to the minority community in addressing the problem of our under representation in the sciences goes back to the 1980s when I was an undergraduate at Hunter College of the City University of New York. At Hunter, I founded a Science Academy for underrepresented minority undergraduates interested in research. We interacted weekly on the research we were doing as part of the MBRS[8] and MARC[9] programs. These synergistic interactions nurtured us as young minority researchers, giving us ownership and confidence in the field as we prepared to become scientists. In fact, our academy was such a success that it even energized the scientific climate among Hunter students in general, especially through our popular speaker series of distinguished scientists.

Without a doubt, an *early* exposure to role models has a positive impact on the type of careers underrepresented minority students choose. It was precisely such experiences that inspired me to go into academia. There is no substitute for the profound impact of testimonials from those who have traversed that road. From my arrival at Duke in 1998, I have mentored numerous Duke minority students and, equally important, have attended their events. I am also honored to have been selected as guest speaker for many minority events (see page 49).

[8]Minority Biomedical Research Support
[9]Minority Access to Research Careers

One of the major events I initiated and organized was the **Duke/Goldman Sachs Diversity Forum** on January 26, 1999. A team of Duke minority alumni working at Goldman Sachs gave testimonials about life in the financial world, including summer internship opportunities and full-time employment. This has now become an annual event at Duke. As a result of the first event, many of our minority students have been hired as summer analysts and Duke was selected as one of the schools eligible for receiving **Goldman Sachs Scholarships for Excellence** for minority students.

I am delighted to report that over ten underrepresented-minority Duke undergraduates have won Goldman Sachs scholarships. Below are *some* of the students:

- **Latarsha Davidson**
- **Edward Errea**
- **Alina Henry**
- **Edward Keith**
- **Deja Lewis**
- **Lauren Moses**
- **Richard Rivera**
- **Catalina Saldarriaga**
- **Kevin Southerland**

I also served as the *Director of the Reginaldo Howard Memorial Scholarship Program* from 2002 to 2003. This scholarship is awarded to the top African-American applicants to Duke's undergraduate program. The recipients routinely rank in the top percentile of their high school classes and demonstrate impressive leadership abilities. Matriculating Reginaldo Howard scholars have gone on to pursue professional and doctoral studies at premier institutions like Duke, and have won prestigious fellowships/scholarships such as the Fulbright, Howard Hughes, Mellon, and Truman.

Overall, I have made myself available to our minority students. Many come to see me for career advice and letters of recommendation. I should add that every Black History Month, I also receive emails from about 100 minority students in middle schools and high schools across the country who are doing projects on African-American mathematicians. They want to know about my research work at Duke, who inspired me when I was their age, how I have dealt with racism, which math courses to take, what kinds of jobs they can get with a math degree, how they can prepare to attend a university like Duke, etc. I believe that these service activities are critical to my community and I do them with great pleasure.

3.2 List of Administrative and Service Experiences

University Service

- Member, Search Committee for Arts & Sciences Dean, Duke University (2015)
- Member, University Priorities Committee, Duke University (2013-2016)
- Chair, Distinguished Professor Nomination Committee in Arts & Sciences, Duke University (2011-2012)
- Member, Distinguished Professor Nomination Committee in Arts & Sciences, Duke University (2010-2012)
- Member, President's Committee on Provost's Review for Renewal, Duke University (2008)
- Member, Nomination Committee, Bass Endowed Chairs, Duke University (2005–2008)
- Pre-Major Adviser, Duke University (2004–2006)
- Panelist, Graduate School Recruitment Weekend for Underrepresented Minority Students, Duke University (October 2004)
- Director, Reginaldo Howard Memorial Scholarship Program, Duke University (2002–2003)
- Member, Arts & Sciences Task Force on the Budget, Duke University (2002)
- Member, Academic Council, Duke University (2002)
- Member, Selection Committee for Duke Endowment Graduate Fellowships, Duke University (2002)
- Member, Selection Committee for A. B. Duke Scholars, Duke University (2002)
- Member, Selection Committee for Residential Life Coordinators, Duke University (spring 2002)
- Presenter, Duke-Up-Close Recruiting Weekend, Duke University (2002)
- Presenter, The Faculty Member as University Citizen,fundraising event, Duke Annual Fund (2002)
- Pre-Major Adviser, Duke University (2001–2002)
- Member, Search Committee for Vice President for Student Affairs, Duke University (2000–2001)
- Member, Search Committee for Assistant Dean for Student Development, Duke University (2000–2001)
- Member, Nomination Committee, Bass Endowed Chairs, Duke University (1999–2002)
- Faculty-in-Residence, Wilson Hall and Bassett Hall, Duke University (1999–2003, 2004–2006)
- Presenter, Arts and Sciences Steering Committee, Duke University (1999)
- Member, Selection Committee for Teaching Awards, Duke University (1999)
- Organized Duke/Goldman Sachs Diversity Forum for Minority Students, Duke University (January 26, 1999)
- Presenter, Duke Regional Campaign Council, fundraising (1999)
- Member, President's Council on Black Affairs, Duke University (1998–2000)
  Co-Chair, Action Committee of the President's Council on Black Affairs, Duke University (1998–1999)
- Academic Adviser, Forbes College, Princeton University (1995–1998)

- Resident Faculty Member, Forbes College, Princeton University (1995–1997)
- Faculty Representative, Alumni Council of Princeton University (1995–1996)
- Martindale Scholarship Selection Committee, Forbes College, Princeton University (1996)
- Summer Scholars Program Lecturer, Princeton University (summers 1996–1997)

Departmental Administrative Service

- Member, Appointments Committee, Duke University, Department of Mathematics (1998–1999, 2001–2002, 2003–2006, 2008–present)
- Member, Search Committee for Professor of the Practice in Mathematics, Duke University, Department of Mathematics (2011–2012)
- Examiner, Qualifying and Preliminary Exams, Duke University, Department of Mathematics (2004–2006)
- Co-Organizer, Einstein Week, Centennial Celebration of Relativity Theory, Duke University, Departments of Mathematics and Physics (Durham, September 5–10, 2005)
- Director of Undergraduate Studies, Duke University, Department of Mathematics (2002)
- Member, Re-Appointment Committee for Assistant Professor of the Practice, Duke University, Department of Mathematics (spring 2002)
- Co-Organize, Seventh Conference for African American Researchers in the Mathematical Sciences, Duke University, Department of Mathematics (Durham-North Carolina, June 22–25, 2001)
- Major Adviser, Duke University, Department of Mathematics (2000–2001, 2003–2006, 2010-2013)
- Organizer, Duke/Goldman Sachs Conference on Mathematical Finance, Duke University, Department of Mathematics (January 26, 1999)
- Examiner, Ph.D. General Exams, Duke University, Department of Mathematics (1999, 2007, 2008)
- Co-Director of Graduate Studies, Princeton University, Department of Mathematics (1996–1998)
- Advanced Placement Officer, Princeton University, Department of Mathematics (1994-1996)
- Examiner, Ph.D. General Exams and Undergraduate Comprehensive Exams, Princeton University, Department of Mathematics (1993–1998)

Departmental Mentor of Junior Faculty

- Carla Cederbaum (Ph.D., Freie Universität, Germany)
  Visiting Assistant Professor, Duke University, Department of Mathematics (2011–2013)
- Amir Aazami (Ph.D., Duke University, U.S.A.)
  Visiting Assistant Professor, Duke University, Department of Mathematics (2011–2013)
- Marcus Werner (Ph.D., University of Cambridge, England)
  Visiting Assistant Professor, Duke University, Department of Mathematics (2009–2011)
- Kumar S. Virbhadra (Ph.D., Physical Research Laboratory, India)
  Postdoctoral Fellow, Duke University, Department of Mathematics (2000–2002)

Departmental Supervisor of Ph.D. Thesis Students

- Marcus Werner, University of Cambridge, Institute of Astronomy (2007–2009, co-adviser)
- Amir Aazami, Duke University, Department of Mathematics (2006–2011)
- Alberto Teguia, Duke University, Department of Mathematics (2005–2011)

Departmental Mentor of Ph.D. Students

- Andrea Watkins, Duke University, Department of Mathematics (2004–2006)
- Nicholas Robbins, Duke University, Department of Mathematics (2003–2005)
- Sean McGee, Duke University, Department of Physics (2003)

Supervisor of M.B.A. Student Research Projects (Fuqua School of Business)

[**1-31**] As part of Fuqua's Executive M.B.A. Finance Concentration Program, I have **supervised 31 finance concentration research projects** (2010-present). These projects covered topics dealing with firm valuation, derivatives, portfolio theory, mergers and acquisitions, etc. The exact titles are not included, but I can provide them upon request.

I also supervised the following research projects as part of the Daytime M.B.A. program at Fuqua:

[**32**] Risk Measures in the Context of Global Financial Crises
Ramona Bejan, Fuqua School of Business, Cross Continent Executive M.B.A. Program (2009)

[**33**] Increasing Technology Awareness in the Belizean Population
Alfredo Frech and Ryan McCoy, Fuqua School of Business, Daytime M.B.A. Program (2009)

[**34**] Ecoresort in Belize: Research Report and Proposals
Siwon Baek, Isao Minegishi, and Travis Peoples, Fuqua School of Business, Daytime M.B.A. Program (2009)

[**35**] The Emerging Oil Industry in Belize
Valentina Nielsen, Fuqua School of Business, Daytime M.B.A. Program (2009)

Supervisor of Undergraduate Research Projects

[**1**] Filtering Financial Time Series using Transforms
Kyuwon Choi, Duke University, Departments of Economics and Mathematics (2011-2012)

[**2**] Carbon Credits
Kyuwon Choi, Duke University, Departments of Economics and Mathematics (2010)

[**3**] Whitney Singularities and Magnification Cross Sections
Luke Stewart, Mellon Fellow, Duke University, Department of Mathematics (2005–2007)

[**4**] Mathematical Pedagogy for STEM Fields
Michelle Sowemimo, Duke University, Department of Psychology (2006–2007)

[**5**] Global Magnification Sum Relations and Resultants
Ibraheem Mohammed, A. B. Duke Scholar, Duke University, Department of Mathematics (2005–2007)

[6] Magnification Probabilities in Microlensing
Wifag Adnan, Duke University, Departments of Economics and Mathematics (2005)

[7] Probability Density for Random Image Magnification
Gabriel Williams, Mellon Fellow, Morehouse College, Department of Mathematics (summer 2005)

[8] Magnification Fractional Differences for a Chang-Refsdal Lens
Peter Blair, Goldwater Scholar, APS Fellow, Mellon Fellow, Duke University, Department of Mathematics (2004–2006)

[9] The Chang-Refsdal Lens: Critical Curves and Caustics
Meredith Houlton, PRUV Research Fellow, Duke University, Department of Mathematics (2002–2003)

[10] Gravitational Microlensing
Jay Strader, Duke University, Department of Physics (2001–2002)

[11] Alternative Price Processes for Black-Scholes: Empirical Evidence and Theory
Samuel Malone, Rhodes Scholar, Goldwater Scholar, Duke Faculty Scholar, PRUV Research Fellow, Duke University, Department of Mathematics/Economics (2000–2002)

[12] Markowitz Portfolio Theory and the Black-Scholes Equation
Paula Sardi, Duke University, Department of Mathematics/Economics (spring 2002)

[13] Electronic Communication Networks for Financial Markets
Vanessa Rousso, Dananberg Fellow, Duke University, Department of Mathematics (fall 2001)

[14] Gravitational Lensing: Center of Light Curves for a Point-Mass Lens System
Paul Bunn, Duke University, Department of Physics (2000–2001)

[15] Mathematical Finance: Portfolio Theory and Option Pricing
Gil Libling, Duke University, Department of Mathematics (2000)

[16] Mathematical Finance: Portfolio Theory and Option Pricing
Joshua Schiffrin, Duke University, Department of Mathematics (2000)

[17] Mathematical Finance: Portfolio Theory and Option Pricing
Shiv Sudhakar, Duke University, Department of Mathematics (2000)

[18] The Morse Inequalities and Gravitational Lensing
Benni Goetz, A.B. Duke Scholar, Duke University, Department of Mathematics (1998–2000)

[19] Computing the Euler Characteristic
Jean Steiner, Princeton University, Department of Mathematics (1996)

[20] Complex Analysis and Gravitational Microlensing
Andre Desssources, University of Massachusetts, Boston, Ronald McNair Program and MIT Minority (1993)

[21] Stability and Genericity of Differentiable Maps
Michelle Wilson, MIT Minority Summer Science Research Program (1991)

[22] Robertson-Walker Universes
Gabriel Reynoso, MIT Minority Summer Science Research Program (1988)

Service to the Discipline

- Member, Board of Trustees, Institute of Pure and Applied Mathematics (2006–2010)
- Member, Board of Governors, Institute of Mathematics and its Applications (2006–2010)
- Member, Selection Committee for Centennial Fellowship, The American Mathematical Society (2004–2006)
- Member, African Scientific Committee for the creation of the African University of Science and Technology (2004–2008)
- Member, Selection Committee for Blackwell-Tapia Prize in Mathematical Science (2004–2006)
- Invited Participant, U.S. National Committee for Mathematics, National Research Council of the National Academy of Sciences (2001)
- Referee for Journals: Astronomy and Astrophysics, Journal of Mathematical Physics, Monthly Notices of the Royal Astronomical Society, The Astrophysical Journal (1998–present)
- Referee for Grant Proposals: National Science Foundation—Geometric Analysis and Gravitational Physics programs (1998–present)

Service to the Underrepresented Minority Community

This list is not exhaustive. It highlights examples showing the variety of service activities.

- **Mentorship (1982–present)**: I have mentored numerous minority students at Hunter College-CUNY (1982–1986), MIT (1986–1988), Princeton University (1993–1998, 2007), and Duke University (1998–present). Received many plaques, including a Service Award from Princetonians of Color Network on February 8, 1996 (*Our Unsung Heroes and Heroines*). I continue to mentor underrepresented minority students from around the country, which has included students at Duke, Harvard, Princeton, and Stanford University.

- **Scholarships (2000–2006)**: As a result of the Duke/Goldman Sachs Diversity Forum I organized in 1999, I developed a relationship between Goldman Sachs and Duke University that resulted in over *ten* Goldman Sachs Scholarships for Duke underrepresented minority students. Some recipients are: Lauren Moses (2000), Catalina Saldarriaga (2000), Latarsha Davidson (2001), Alina Henry (2001), Deja Lewis (2001), Richard Rivera (2002), Edward Errea (2002), Edward Keith (2003).

- **Director, Reginaldo Howard Memorial Scholarship Program (2002–2003)**: Scholarship program at Duke aimed at address the issue of underrepresentation among minorities. Students receive a full scholarship to attend Duke.

- **Supervisor of Research Projects, Science Program for Underpresented Minorities, MIT (1988–1995):**
  - Michele Wilson, Department of Mathematics, MIT (MIT Summer Science Research Program 1991, MIT graduate student 1991–1995)
  - Andre Dessources, undergraduate (Ronald McNair and MIT Summer Science Research Programs, spring and summer 1993)
  - Ivon Smikle, undergraduate (Ronald McNair Program, University of Massachusetts, Boston; fall 1992, spring 1993)
  - Gabriel Reynoso, undergraduate (MIT Summer Science Research Program, 1988)
- **Guest Speaker and Panelist**:[10]
  - Guest Speaker, Underrepresented Minority Students, North Park University (October 2008)
  - Guest Speaker, Underrepresented Minority Students at Honors College, Johnson C. Smith University—an HBCU (September 2008)
  - Guest Speaker, MBRS-MARC Program Underrepresented Minority Students, Hunter College, CUNY (September 2008)
  - Guest Speaker, National Society of Black Engineers, Princeton University (October 2008)
  - Panelist, Recruiting Underrepresented Minorities for Careers in the Mathematical Sciences, Boston University, Department of Mathematics (April 2006)
  - Guest Speaker, African-American Mathematics Symposium, Boston University, Department of Mathematics (April 2006)
  - Guest Speaker, National Institute of Science Beta Kappa Chi Awards Banquet, Montgomery, Alabama (March 2006)
  - Guest Speaker, HBCU-UP Conference, Baltimore, Maryland (February 2006)
  - Panelist, Mellon Mays Undergraduate Fellowship Fellowship Southern Regional Conference, Rice University, Houston, Texas (October 2005)
  - Guest Speaker, Mathematical Association of America Conference, University of Texas,Arlington (April 2005)
  - Guest Speaker, Marjorie Lee Brown Colloquium, University of Michigan, Department of Mathematics (In Celebration of Martin Luther King, Jr. Day, January 2004)
  - Panelist, African American Men in the Academy, University of North Carolina (Chapel Hill) Summer Pre-Graduate Program for Underrepresented Minorities (June 2004)
  - Guest Speaker, Lessons from My Academic Journey, Black Graduate Student Association of MIT (February 2004)
  - Guest Speaker, Corey T. Williams Banquet and Ball for minority undergraduates, Duke University (February 2002)
  - Keynote Presenter, Recommendation Paper on Education to Prime Minister of Belize, Belizean Professorship Program, Belize (May 2000)
  - Guest Speaker, Julian Abele Awards Banquet for minority graduate and professional students, Duke University (March 2000)

[10] This list is not exhaustive.

- Guest Speaker, Cook Society, Duke University (fall 1998)
- Guest Speaker, NSBE Awards Banquet, Princeton University (spring 1997)
- Guest Presenter, Princeton's Summer Program for Minority Students (summer 1995)
- Guest Speaker, Sixth Annual Career Day, West Athens Elementary School, Los Angeles (spring 1995) Received Community Service Award.
- Panelist, Princeton University African American and Latino Open House (fall 1994)
- Panelist, First Invitational Workshop on MIT Leadership for Diversity (fall 1994)
- Lecture, Princeton Chapter of National Society of Black Engineers (spring 1994)
- Lecture, Princeton Summer Program for Minority Students (summer 1994)

4 International Experience: Developing World

My service to society in the U.S. has been marked by over two decades of work in addressing the problem of the underrepresentation of minorities in mathematics and the sciences; see pages 48-50. As a proud naturalized U.S. citizen from an underrepresented minority group, it has brought me great personal contentment to give back to our society in this important way. Alongside these efforts, I have also reached out to the developing world.

4.1 New University in Africa

I served as a member (2004–2008) of the African Scientific Committee involved with creating a new university in Africa: **The African University of Science and Technology** (AUST) in Abuja, Nigeria. The committee was charged with designing a university wide curriculum in science and engineering that meet world-class standards.

The AUST-Abuja was established in 2007 by the Nelson Mandela Institution for Knowledge Building and the Advancement of Science and Technology in Sub-Saharan African and with the incubating support of the World Bank.

The mission of AUST is to:

> "... advance knowledge and educate students in science, technology, and other areas of scholarship that will best serve the African continent in the 21st century. AUST is dedicated to providing its students with an education that combines rigorous academic study and the excitement of discovery with the support and intellectual stimulation of a diverse international and diaspora community. We seek to develop in each member of the AUST community the ability and passion to work wisely, creatively, and effectively for the betterment of humankind."

The long-term goal of AUST has been to build several other campuses across sub-Saharan Africa to serve as catalysts for developing the African human capital in science and technology.

More about the AUST can be found at **www.aust-abuja.org**.

4.2 New Institute in Belize

Building on the lessons learned from my work in the U.S. with underrepresented minorities in mathematics and science, and in Africa with the AUST, I founded in 2005 the **Petters Research Institute**[11] (PRI) in Dangriga, Belize, as a way of giving back to my place of birth.

PRI is a non-profit, private, politically-neutral institute charged with:

- creating an interdisciplinary center of excellence in the application, theory, and pedagogy of STEM fields, as well as their synergistic interactions with business administration and entrepreneurship to induce economic growth

[11] Named in honor of my family.

- promoting outreach programs to scholars and professionals outside science, and to the community at large, encouraging collaboration and mutual understanding to create synergies for the betterment of the Belizean people.

Activities at PRI have included research in mathematics and science, summer academies for students, workshops on pedagogical tools for teachers, and collaborations with the Belize Defence Force to train soldiers in mathematics and science. The institute also offers scholarships to needy students.

PRI pursues its mission through programs that include (but are not limited to):

- advancing a technology sector in the Belizean economy by developing the Belizean human capital in STEM fields, and tying these fields intimately with the needs of businesses and entrepreneurs to create economic growth

- bringing together world-class researchers to generate practical findings and policy recommendations that ramify into the Belizean educational, technological, business, and health sectors

- partnering with primary, secondary, and tertiary academic institutions to offer enhanced e-learning environments and opportunities for students, educators, and the community

- developing ties with U.S. educational institutions and businesses to promote human capital development and cross-national job opportunities, particularly for needy underrepresented minorities and non-minorities from economically challenged communities.

In addition, through my appointments at Duke in the Department of Mathematics, Department of Physics, and the Fuqua School of Business, I have already begun synergistic scientific and entrepreneurial interactions between Duke students and PRI, for example, scientific workshops, technology business planning, Garifuna tourist village feasibility study, ecoresort and green housing planning, etc.

An example of one of our summer programs, **Biodiversity and DNA Barcoding**, which was led by researchers from the City University of New York Graduate Center, can be found at **www.biobelize.org**.

Visit **www.pribelize.org** for more information on PRI.

4.3 Belize Prime Minister's Council of Science Advisers

I am the inaugural Chairman of the Council of Science Advisers to the Prime Minister of Belize (PM-CSA). The PM-CSA was commissioned on March 29, 2010 and is charged with directly advising the Prime Minister and his Cabinet on furthering the development of Belize through practical and environmentally sustainable applications of Science, Technology, Engineering, and Mathematics to create economic growth, strengthen education, enhance health, and fortify Belize's national security. The council will provide the Prime Minister and his office, and Cabinet with independent, politically-neutral counsel on policies, proposals, and any issues that the Prime Minister and his office deem appropriate.

The activities of the PM-CSA will include assisting the Prime Minister with:

- Framing science policies that convert into practical programs that promote economic growth.

- Attracting science and technology investors and industries, particularly in green technologies.
- Identifying strategic economic opportunities that cut across different government ministries as well as creating synergistic opportunities for ministries using technology.
- Developing the Belizean human capital in STEM fields with emphasis on applications that advance national development.
- Reviewing policy proposals for scientific soundness.

An initial aim of the PM-CSA is to assist the Prime Minister with formulating a national green policy that lays a path for Belize to grow economically by striving to become one of the world's foremost developing nations in green technology. Specifically, the PM-CSA will advise the Prime Minister on practical measures for how to:

- Formulate a national development strategy that builds upon Belize's track record in ecotourism and environmental conservation to forge new economic growth through green technologies.
- Attract investors and innovators in renewable energy, recycling, waste management, and green construction to create new jobs, achieve consumer energy savings, and promote synergies that enhance ecotourism.
- Systematize, inventory, and manage the new economic value available to Belize through carbon sequestration credits from existing forest reserves and their replenishing, and the carbon offset credits realized from the introduction of renewable energy technologies.
- Advance purposefully computer technology, particularly software development and IT service, as an additional source of economic growth.

More about the PM-CSA can be found at **www.pribelize.org**.