We will have a make-up class on Monday February 2, 1:25–2:40, in Bio Sci 063 (same room as last time). My plan for that class is to finish discussing Möbius transformations (Chapter 3 sections 3.3, 3.4, 3.5).

1. For each part, find an analytic function mapping:
   (a) the open unit disk \(|z| < 1\) onto the punctured disk \(0 < |z| < 1\);
   (b) the region between the circles \(|z| = 2\) and \(|z - 1| = 1\) (not including the circles themselves) one-to-one and onto the open unit disk;
   (c) \(\mathbb{C} \setminus [-1, 1]\) onto the open unit disk.
   
   Hint: exponentials, logarithms, and Möbius transformations may all help with constructing these functions.

2. Find, with proof, all Möbius transformations that send the unit circle \(|z| = 1\) to itself. (Hint: the function \(\varphi_a\) defined by \(\varphi_a(z) = \frac{z-a}{1-\overline{a}z}\) may be of interest.)

3. Ahlfors section 2.1.4 exercise 4, p. 33. Hint: if \(r(z)\) is a rational function, consider \(\overline{r(z^{-1})}\).

4. Ahlfors section 3.3.2 exercise 3, p. 80.