Math 621 Homework 1—due Friday January 26

Spring 2018

Changes to class dates in the near future:

- $+$: we will have class **Monday 1/22** and **Monday 2/5**, both at 3:05–4:20 in the usual room.
- $-$: we will not have class *Wednesday 1/31*.

For a full updated schedule of class changes, please see the course web site, `https://services.math.duke.edu/~ng/math621/`.

1. Let $\mathbb{CP}^n$ denote the usual complex projective n-space, defined as the quotient of $\mathbb{C}^{n+1} \setminus \{0\}$ by the group $\mathbb{C} \setminus \{0\}$ acting by scalar multiplication. Show that $\mathbb{CP}^n$ is a smooth manifold of dimension $2n$ by constructing an atlas $\{(f_i, U_i, V_i)\}$ and checking that the transition functions $f_j^{-1} \circ f_i$ (mapping what subset of $\mathbb{R}^{2n}$ to what subset of $\mathbb{R}^{2n}$?) are smooth.
2. do Carmo chapter 0 exercise 5, p. 32.
3. do Carmo chapter 0 exercise 9, p. 33, but don't do the Klein bottle case in (b).
4. (a) In #3, you showed that $\mathbb{RP}^n$ is orientable if and only if n is odd. In this problem, prove directly that $\mathbb{RP}^n$ is orientable if n is odd by explicitly giving an oriented atlas for $\mathbb{RP}^n$, along the lines of the atlas given in class and on pp. 4–5, and proving that your atlas is oriented.

   (b) Use your answer to #1 to prove that $\mathbb{CP}^n$ is orientable for all n.