5. Let \((K^{\star, \star}, \delta, d)\) be the double complex of \(\mathbb{R}\)-vector spaces defined as follows. A basis for \(K^{\star, \star}\) is given by 13 generators \(v^{0,1}, v^{0,2}, v^{1,0}, v^{1,1}, v^{2,1}, v^{2,0}, v^{1,2}, v^{1,1}, v^{3,1}, v^{1,2}, v^{2,0}, v^{2,1}, v^{3,1}, v^{2,2}\), where the superscript denotes the bigrading; e.g., \(K^{1,0}\) is the 2-dimensional vector space over \(\mathbb{R}\) generated by \(v^{1,0}_1\) and \(v^{1,0}_2\). The differentials are given by
\[
\delta(v^{0,1}) = v^{1,1}_3, \quad \delta(v^{0,2}) = v^{2,0}_3, \quad \delta(v^{1,1}) = v^{2,1}_1, \quad \delta(v^{2,0}) = v^{2,1}_{1+3}, \quad \delta(v^{1,2}) = v^{2,2}
\]
and \(\delta = 0\) on all other generators;
\[
d(v^{0,1}) = v^{0,2}_3, \quad d(v^{1,0}) = v^{1,1}_3, \quad d(v^{1,1}) = v^{1,1}_1, \quad d(v^{2,0}) = v^{2,1}_1, \quad d(v^{2,1}) = v^{2,2}
\]
and \(d = 0\) on all other generators.

(a) Calculate the (singly) graded vector space \(H^\star(K, D)\) by calculating the spectral sequence \(\{(E^r_{\star, \star}, d^r_{\star, \star})\}\) for \(K\) with the usual filtration.

(b) Recalculate \(H^\star(K, D)\) by calculating the alternate spectral sequence \(\{(E'_r, d'_r)\}\) obtained by switching the role of rows and columns \((E'_1 = H^{\star, \star}(K, \delta)\) and \(d'_r\) has bidegree \((1-r, r))\).

6. (a) For arbitrary \(r \geq 1\), give an example of a finite-dimensional double complex of \(\mathbb{R}\)-vector spaces for which \(d_r\) is nonzero.

(b) The Lie group \(SO(3)\) is diffeomorphic to \(\mathbb{R}\mathbb{P}^3\). Use the fiber bundle \(SO(3) \to SO(4) \to S^3\) to calculate \(H^\star_{DR}(SO(4))\) as a graded vector space.