4. Call an open cover $\mathcal{U}$ of a smooth $n$-manifold $M$ decent if all intersections $U_{\alpha_0 \ldots \alpha_k}$ for $k \geq 0$ are diffeomorphic to a disjoint union of some (possibly zero) number of copies of $\mathbb{R}^n$.

(a) Explain why the proof that

$$H^*_{DR}(M) \cong \check{H}^*(\mathcal{U}, \mathbb{R})$$

for good covers extends to decent covers as well.

(b) Find a three-set decent cover for $\mathbb{R}P^2$ and use it to calculate $H^*_{DR}(\mathbb{R}P^2)$.

(c) Find a three-set decent cover for $T^2$ and use it to calculate $H^*_{DR}(T^2)$.

(d) Find, with proof, a decent cover for $S^2$ with the smallest possible number of sets, and use it to calculate $H^*_{DR}(S^2)$. 