1. Munkres 2.16 #10, p. 92.

2. Let $\mathbb{Z}_+$ denote the set of natural numbers (positive integers). Consider the sets

\[ S_+ = \{1/n : n \in \mathbb{Z}_+\} \]
\[ S_- = \{-1/n : n \in \mathbb{Z}_+\} \]

as subsets of $\mathbb{R}$. Find all limit points for each of $S_+$ and $S_-$ in each of the following topologies on $\mathbb{R}$: the discrete topology; the indiscrete topology; the standard topology; the lower-limit topology.

3. Let $X$ be a set with the finite-complement topology. Consider a sequence $\{x_i\}_{i=1}^\infty = \{x_1, x_2, \ldots\}$ of elements of $X$, and write

\[ S = \bigcup_{i=1}^\infty \{x_i\} = \{x : x = x_i \text{ for some } i\} \subset X.\]

(a) If $S$ is an infinite set, and any particular element of $X$ appears in the sequence $\{x_1, x_2, \ldots\}$ at most finitely many times, prove that the sequence $\{x_i\}$ converges to $x$ for any $x \in X$.

(b) If $S$ is a finite set, find a condition on a point $x \in X$ that is both necessary and sufficient to ensure that the sequence $\{x_i\}$ converges to $x$. (Your answer should involve $x$ and the sequence $x_i$, but obviously it should be less vacuously true than "$\{x_i\}$ converges to $x$".)


5. (a) Munkres 2.17 #13, p. 101.

   (b) Munkres 2.17 #15, p. 101.


*Fun optional problem (not to turn in), if you’re looking for a challenge: Munkres 2.17 #21, p. 102.*