1. (a) 4: \[ \text{Shut } = \text{(nonempty open set)} \]

(b) 29: Break these up according to how many 1-element open subsets there are: call this number \( n \).

\[ n=0: \]

\[ n=1: \]

\[ n=2: \]

\[ n=3: \]

2. (a) No. \( X=\mathbb{R}, \) standard topology, \( Y = U = [a,b) \)

(b) Yes. \( U \) open in \( Y \) \( \Rightarrow \) there is an open \( V \) in \( X \) with \( U = V \cap Y \). Since \( V, Y \) are both open in \( X \), so is \( U = V \cap Y \).
3. If $i=2$, then regardless of the topology on $X_j$, if $U \subseteq X_j$ is open then $f^{-1}(U) \subseteq X_2$ is open since every subset is open.

Thus for $i=2$, \[ C(X_2, X_j) = \{ \text{all functions } f: \mathbb{R} \to \mathbb{R} \} \]
for all of $j=1, 2, 3$.

If $j=3$, then note that for any $f: \mathbb{R} \to \mathbb{R}$, $f^{-1}(\emptyset) = \emptyset$ and $f^{-1}(\mathbb{R}) = \mathbb{R}$.

Both of these are open in $X_i$ regardless of $i$.

Thus for $j=3$, \[ C(X_i, X_3) = \{ \text{all functions } f: \mathbb{R} \to \mathbb{R} \} \]
for all of $i=1, 2, 3$.

Finally, consider the cases $(i, j) = (3, 1)$ and $(3, 2)$. We claim \[ C(X_3, X_j) = \{ \text{constant functions} \} \]
for $j=1$ or $j=2$.

First suppose $f: \mathbb{R} \to \mathbb{R}$ is constant, $f(x) = c \forall x \in \mathbb{R}$.
Then for any $U \subseteq \mathbb{R} = X_j$, $f^{-1}(U) = \begin{cases} \emptyset & \text{if } c \notin U, \\ \mathbb{R} & \text{if } c \in U \end{cases}$.

In all cases $f^{-1}(U)$ is open in $X_3$. Thus constant functions are continuous as maps $X_3 \to X_j$.

Now suppose $f: \mathbb{R} \to \mathbb{R}$ is not constant. Then there exist $x_1, x_2 \in \mathbb{R}$ with $f(x_1) = y_1$, $f(x_2) = y_2$, and $y_1 \neq y_2$. Choose an open set $U$ in $X_j$ with $y_1 \in U$ and $y_2 \notin U$. (For the discrete topology $j=2$, we can choose $U = \{ y_1 \}$; for the standard topology $j=1$, choose $U$ to be some open interval containing $y_1$ but not $y_2$.) Then $f^{-1}(U)$ contains $x_1$ and does not contain $x_2$, so $f^{-1}(U) \neq \emptyset$ and $f^{-1}(U) \neq \mathbb{R}$, that is, $f^{-1}(U)$ is not open.

Thus if $f$ is not constant, then it is not continuous as a map $X_3 \to X_j$, $j=1, 2$. 