Math 103X.02 Homework 6 Answers & Solutions

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§5.1, 6: $(\ln 9)/4$; 14: The region is bounded by $z = |x|\sin(\pi y)$, the xy-plane ($z = 0$), and the planes $x = -2$, $x = 3$, $y = 0$, and $y = 1$. The volume of this region is $13/\pi$.

§5.2, 20: (a) The volume of a solid prism with constant height is the area of the base times the height; in this case the height is 1. A more formal proof can be constructed using Riemann sums. (b) If D is the region bounded by the circle $x^2 + y^2 = a^2$, then

$$\iint_D 1\,dA = \int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} dy\,dx = \int_{-a}^{a} 2\sqrt{a^2-x^2}\,dx = \pi a^2.$$

Extra problems:

1. (a) The volume of the hemisphere is

$$\begin{aligned}
\iint_D \sqrt{r^2-x^2-y^2}\,dA &= \int_{-r}^{r}\int_{-\sqrt{r^2-x^2}}^{\sqrt{r^2-x^2}} \sqrt{r^2-x^2-y^2}\,dy\,dx \\
&= \int_{-r}^{r}\left(\frac{y\sqrt{r^2-x^2-y^2}}{2} + \frac{r^2-x^2}{2}\sin^{-1}\frac{y}{\sqrt{r^2-x^2}}\right)\Bigg|_{-\sqrt{r^2-x^2}}^{\sqrt{r^2-x^2}} dx \\
&= \int_{-r}^{r}\frac{\pi}{2}(r^2-x^2)\,dx \\
&= \frac{2\pi r^3}{3}.
\end{aligned}$$

   (b) $4\pi r^3/3$.

2. (a) The disk is centered at $(x_0, 0, 0)$, and $(x_0, \sqrt{r^2-x_0^2}, 0)$ is a point on the boundary of the disk. It follows that the radius of the disk is the distance between these two points, or $\sqrt{r^2-x_0^2}$. The area is $\pi(r^2-x_0^2)$.

   (b) The slices go from $x = -r$ to $x = r$; thus the volume is

$$\int_{-r}^{r}\pi(r^2-x^2)\,dx = \frac{4\pi r^3}{3}.$$