§4.1: 8. \( p_1(x, y) = 1, p_2(x, y) = 1 - x^2 - y^2; \)

14. \( Hf(0, 0) = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}. \)

§4.2, 32. There is one interior critical point at \((0, 1/2)\), and four boundary critical points at \((0, 1), (0, -1), (\sqrt{3}/2, -1/2), \) and \((-\sqrt{3}/2, -1/2)\). The coldest spot on the plate is \((0, 1/2)\) where the temperature is 11/4, and the hottest spots on the plate are \((\pm\sqrt{3}/2, -1/2)\) where the temperature is 21/4.

§4.2, 34. The absolute minimum, \(-2\), occurs at \((\pi, 0)\), and the absolute maximum, 5, occurs at \((0, \pi/2)\).

§4.2, 46(b). The only critical point for \( f \) is at \((0, 1)\) and \( f(0, 1) = 1; \) the second derivative test shows that \((0, 1)\) is a local maximum. On the other hand, \( f(0, y) = 3y - 1 - y^3\), so as \( y \to -\infty, f(0, y) \to +\infty. \) Thus \( f(0, 1) = 1 \) is not a global maximum.

§4.2, 47(b). The two critical points of \( f \), both local maxima, are \((2, 1)\) and \((0, -1)\).

§4.3, 18. Maximum and minimum are both attained by the extreme value theorem since the sphere is compact. The maximum is \( 9\sqrt{3} \) at \((3\sqrt{3}, 3\sqrt{3}, -3\sqrt{3})\) and the minimum is \(-9\sqrt{3} \) at \((-3\sqrt{3}, -3\sqrt{3}, 3\sqrt{3})\).

§4.3, 22. Radius 6 feet, height 21 feet; 28. Nearest point \((1, 1, 2)\), farthest point \((-2, -2, 8)\).

§4.3, 31(b). The set \( \{(x, y) | xy = 6\} \) is not connected; there’s a component with \( x, y > 0 \) and one with \( x, y < 0 \). Although \( f(x, y) \) attains a local minimum of \( 2\sqrt{6} \) on the first component at \((\sqrt{6}, \sqrt{6})\), it is strictly smaller than that on the entire second component; although \( f(x, y) \) attains a local maximum of \(-2\sqrt{6} \) on the second component at \((-\sqrt{6}, -\sqrt{6})\), it is strictly larger than that on the entire first component.

§4.6, 20(a). \( \frac{a}{v_1 \cos \theta_1} + \frac{b}{v_2 \cos \theta_2}. \)