Math 103X.02 Homework 4—due October 6

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§2.5: 1, 3, 8 (I assume "your son" = the child in question!), 11, 19, 26, 28, 29, 31, 33, 35

§2.6: 3, 7, 12, 15, 17, 23, 33, 41(a) and 41(b)

§2.8: 30

§4.1: 27

Extra problems:

1. (a) Let $S_1 \subset \mathbb{R}^3$ denote the surface $F_1(x, y, z) = c_1$ and let $S_2 \subset \mathbb{R}^3$ denote the surface $F_2(x, y, z) = c_2$, where F_1 and F_2 are scalar valued functions and c_1 and c_2 are constants. Suppose that S_1 and S_2 intersect in a (1-dimensional) curve C, and that (x, y, z) is a point on this curve. Show that the vector $\vec{\nabla} F_1(x, y, z) \times \vec{\nabla} F_2(x, y, z)$ is tangent to C at the point (x, y, z).

   (b) Let S_1 denote the surface $z = 7x^2 - 12x - 5y^2$, let S_2 denote the surface $xyz^2 = 2$, and let C denote the curve which is the intersection of S_1 and S_2. Find a parametric equation for the tangent line to C at the point $(2, 1, -1)$.

   (c) (Colley §2.6 # 24) Show that S_1 and S_2 from (b) intersect orthogonally at the point $(2, 1, -1)$. (Hint: calculate the angle between the normal vectors to S_1 and S_2 at that point.)

2. Suppose that (x_0, y_0) is a point on the curve in $\mathbb{R}^2$ defined by the equation $xy^2 + 3xy + 5y + x = 0$.

   (a) Using the Implicit Function Theorem, find which point(s) (x_0, y_0) satisfy the following condition: near (x_0, y_0), the curve defined by $xy^2 + 3xy + 5y + x = 0$ *cannot* be described as the graph of some function $y = f(x)$.

   (b) Check your answer to (a) directly, using the quadratic formula.

Note: §2.5 # 26(a) and 28(a) (implicit partial differentiation) are important results which you may be responsible for knowing on future tests.

If you're not overly familiar with matrix multiplication, I would suggest working through §1.6 # 17 and 18, though you don't need to turn in these problems. The answer to # 18 is

$$\begin{bmatrix} -4 & 9 & 5 \\ -8 & 9 & 10 \end{bmatrix}.$$