LIST OF PUBLICATIONS, WITH ABSTRACTS

LENHARD NG


  Abstract: We show that the set of augmentations of the Chekanov–Eliashberg algebra of a Legendrian link underlies the structure of a unital A-infinity category. This differs from the non-unital category constructed by Bourgeois and Chantraine, but is related to it in the same way that cohomology is related to compactly supported cohomology. The existence of such a category was predicted by Shende, Treumann, and Zaslow, who moreover conjectured its equivalence to a category of sheaves on the front plane with singular support meeting infinity in the knot. After showing that the augmentation category forms a sheaf over the x-line, we are able to prove this conjecture by calculating both categories on thin slices of the front plane. In particular, we conclude that every augmentation comes from geometry.

• Obstructions to Lagrangian concordance (with C. Cornwell and S. Sivek), submitted, arXiv:1411.1364.

  Abstract: We investigate the question of the existence of a Lagrangian concordance between two Legendrian knots in \( \mathbb{R}^3 \). In particular, we give obstructions to a concordance from an arbitrary knot to the standard Legendrian unknot, in terms of normal rulings. We also place strong restrictions on knots that have concordances both to and from the unknot and construct an infinite family of knots with non-reversible concordances from the unknot. Finally, we use our obstructions to present a complete list of knots with up to 14 crossings that have Legendrian representatives that are Lagrangian slice.


  Abstract: We give a combinatorial description of the Legendrian contact homology algebra associated to a Legendrian link in \( S^1 \times S^2 \) or any connected sum \( \#^k(S^1 \times S^2) \), viewed as the contact boundary of the Weinstein manifold obtained by attaching 1-handles to the 4-ball. In view of the surgery formula for symplectic homology, this gives a combinatorial description of the symplectic homology of any 4-dimensional Weinstein manifold (and of the linearized contact homology of its boundary). We also study examples and discuss the invariance of the Legendrian homology algebra under deformations, from both the combinatorial and the analytical perspectives.

Abstract: We study the connection between topological strings and contact homology recently proposed in the context of knot invariants. In particular, we establish the proposed relation between the Gromov-Witten disk amplitudes of a Lagrangian associated to a knot and augmentations of its contact homology algebra. This also implies the equality between the $Q$-deformed $A$-polynomial and the augmentation polynomial of knot contact homology (in the irreducible case). We also generalize this relation to the case of links and to higher rank representations for knots. The generalization involves a study of the quantum moduli space of special Lagrangian branes with higher Betti numbers probing the Calabi-Yau. This leads to an extension of SYZ, and a new notion of mirror symmetry, involving higher dimensional mirrors. The mirror theory is a topological string, related to D-modules, which we call the “D-model”. In the present setting, the mirror manifold is the augmentation variety of the link. Connecting further to contact geometry, we study intersection properties of branches of the augmentation variety guided by the relation to D-modules. This study leads us to propose concrete geometric constructions of Lagrangian fillings for links. We also relate the augmentation variety with the large $N$ limit of the colored HOMFLY, which we conjecture to be related to a $Q$-deformation of the extension of $A$-polynomials associated with the link complement.


Abstract: O. Plamenevskaya associated to each transverse knot $K$ an element of the Khovanov homology of $K$. In this paper, we give two refinements of Plamenevskaya’s invariant, one valued in Bar-Natan’s deformation of the Khovanov complex and another as a cohomotopy element of the Khovanov spectrum. We show that the first of these refinements is invariant under negative flypes and $SZ$ moves; this implies that Plamenevskaya’s class is also invariant under these moves. We go on to show that for small-crossing transverse knots $K$, both refinements are determined by the classical invariants of $K$.


Abstract: This is a survey of knot contact homology, with an emphasis on topological, algebraic, and combinatorial aspects.


Abstract: We study satellites of Legendrian knots in $\mathbb{R}^3$ and their relation to the Chekanov–Eliashberg differential graded algebra of the knot. In particular, we generalize the well-known correspondence between rulings of a Legendrian knot in $\mathbb{R}^3$ and augmentations of its DGA by showing that the DGA has finite-dimensional representations if and only if there exist certain rulings of satellites of the knot. We derive several consequences of this result, notably that the question of existence of ungraded finite-dimensional representations for the DGA of a Legendrian knot depends only on the topological type and Thurston–Bennequin number of the knot.

Abstract: The conormal lift of a link $K$ in $\mathbb{R}^3$ is a Legendrian submanifold $\Lambda_K$ in the unit cotangent bundle $U^*\mathbb{R}^3$ of $\mathbb{R}^3$ with contact structure equal to the kernel of the Liouville form. Knot contact homology, a topological link invariant of $K$, is defined as the Legendrian homology of $\Lambda_K$, the homology of a differential graded algebra generated by Reeb chords whose differential counts holomorphic disks in the symplectization $\mathbb{R} \times U^*\mathbb{R}^3$ with Lagrangian boundary condition $\mathbb{R} \times \Lambda_K$.

We perform an explicit and complete computation of the Legendrian homology of $\Lambda_K$ for arbitrary links $K$ in terms of a braid presentation of $K$, confirming a conjecture that this invariant agrees with a previously-defined combinatorial version of knot contact homology. The computation uses a double degeneration: the braid degenerates toward a multiple cover of the unknot which in turn degenerates to a point. Under the first degeneration, holomorphic disks converge to gradient flow trees with quantum corrections. The combined degenerations give rise to a new generalization of flow trees called multiscale flow trees. The theory of multiscale flow trees is the key tool in our computation and is already proving to be useful for other computations as well.


Abstract: We present an atlas of Legendrian knots in standard contact three-space. This gives a conjectural Legendrian classification for all knots with arc index at most 9, including alternating knots through 7 crossings and nonalternating knots through 9 crossings. Our method involves a computer search of grid diagrams and applies to transverse knots as well. The atlas incorporates a number of new, small examples of phenomena such as transverse nonsimplicity and non-maximal non-destabilizable Legendrian knots, and gives rise to new infinite families of transversely nonsimple knots.


Abstract: We give a combinatorial treatment of transverse homology, a new invariant of transverse knots that is an extension of knot contact homology. The theory comes in several flavors, including one that is an invariant of topological knots and produces a three-variable knot polynomial related to the $A$-polynomial. We provide a number of computations of transverse homology that demonstrate its effectiveness in distinguishing transverse knots, including knots that cannot be distinguished by the Heegaard Floer transverse invariants or other previous invariants.


Abstract: We construct a new invariant of transverse links in the standard contact structure on $\mathbb{R}^3$. This invariant is a doubly filtered version of the knot contact homology differential graded algebra (DGA) of the link. Here the knot contact homology of a link in $\mathbb{R}^3$ is the Legendrian contact homology DGA of its conormal lift into the unit cotangent bundle.
$S^*\mathbb{R}^3$ of $\mathbb{R}^3$, and the filtrations are constructed by counting intersections of the holomorphic disks of the DGA differential with two conormal lifts of the contact structure. We also present a combinatorial formula for the filtered DGA in terms of braid representatives of transverse links and apply it to show that the new invariant is independent of previously known invariants of transverse links.


Abstract: In 1997, Chekanov gave the first example of a Legendrian nonsimple knot type: the $m(5_2)$ knot. Epstein, Fuchs, and Meyer extended his result by showing that there are at least $n$ different Legendrian representatives of the $m((2n + 1)_2)$ knot with maximal Thurston–Bennequin number. In this paper we give a complete classification of Legendrian and transverse representatives of twist knots. In particular, we show that the $m((2n + 1)_2)$ knot has exactly $\lceil \frac{n^2}{2} \rceil$ Legendrian representatives with maximal Thurston–Bennequin number, and $\lceil \frac{n}{2} \rceil$ transverse representatives with maximal self-linking number. Our techniques include convex surface theory, Legendrian ruling invariants, and Heegaard Floer homology.


Abstract: We use grid diagrams to present a unified picture of braids, Legendrian knots, and transverse knots.


Abstract: We construct a combinatorial invariant of Legendrian knots in standard contact three-space. This invariant, which encodes rational relative Symplectic Field Theory and extends contact homology, counts holomorphic disks with an arbitrary number of positive punctures. The construction uses ideas from string topology.


Abstract: We apply knot Floer homology to exhibit an infinite family of transversely nonsimple prime knots starting with $10_{132}$. We also discuss the combinatorial relationship between grid diagrams, braids, and Legendrian and transverse knots in standard contact $\mathbb{R}^3$.


Abstract: We give a simple unified proof for several disparate bounds on Thurston–Bennequin number for Legendrian knots and self-linking number for transverse knots in $\mathbb{R}^3$, and provide a template for possible future bounds. As an application, we give sufficient conditions for some of these bounds to be sharp.
• Transverse knots distinguished by knot Floer homology (with P. Ozsváth and D. Thurston), *J. Symplectic Geom.* **6** (2008), no. 4, 461–490.

Abstract: We exhibit pairs of transverse knots with the same self-linking number that are not transversely isotopic, using the recently defined knot Floer homology invariant for transverse knots and some algebraic refinements of it.

• On arc index and maximal Thurston–Bennequin number, submitted; math.GT/0612356.

Abstract: We discuss the relation between arc index, maximal Thurston–Bennequin number, and Khovanov homology for knots. As a consequence, we calculate the arc index and maximal Thurston–Bennequin number for all knots with at most 11 crossings. For some of these knots, the calculation requires a consideration of cables which also allows us to compute the maximal self-linking number for all knots with at most 11 crossings.


Abstract: We extend knot contact homology to a theory over the ring $\mathbb{Z}[\lambda^{\pm 1}, \mu^{\pm 1}]$, with the invariant given topologically and combinatorially. The improved invariant, which is defined for framed knots in $S^3$, can distinguish many pairs of knots, including mutants, and can also be defined for knots in arbitrary manifolds. It contains the Alexander polynomial and naturally produces a two-variable polynomial knot invariant which is related to the $A$-polynomial.


Abstract: We summarize recent work on a combinatorial knot invariant called knot contact homology. We also discuss the origins of this invariant in symplectic topology, via holomorphic curves and a conormal bundle naturally associated to the knot.


Abstract: We strengthen the link between holomorphic and generating-function invariants of Legendrian knots by establishing a formula relating the number of augmentations of a knot’s contact homology to the complete ruling invariant of Chekanov and Pushkar.


Abstract: We apply contact homology to obtain new results in the problem of distinguishing immersed plane curves without dangerous self-tangencies.


Abstract: We establish an upper bound for the Thurston–Bennequin number of a Legendrian link using the Khovanov homology of the underlying topological link. This bound is sharp in particular for all alternating links, and knots with nine or fewer crossings.

Abstract: Differential graded algebra invariants are constructed for Legendrian links in the 1-jet space of the circle. In parallel to the theory for $\mathbb{R}^3$, Poincaré–Chekanov polynomials and characteristic algebras can be associated to such links. The theory is applied to distinguish various knots, as well as links that are closures of Legendrian versions of rational tangles. For a large number of two-component links, the Poincaré–Chekanov polynomials agree with the polynomials defined through the theory of generating functions. Examples are given of knots and links which differ by an even number of horizontal flypes that have the same polynomials but distinct characteristic algebras. Results obtainable from a Legendrian satellite construction are compared to results obtainable from the DGA and generating function techniques.


Abstract: We present a topological interpretation of knot and braid contact homology in degree zero, in terms of cords and skein relations. This interpretation allows us to extend the knot invariant to embedded graphs and some higher-dimensional knots. We give a related presentation for knot contact homology in terms of plats, including a calculation for all two-bridge knots.


Abstract: We introduce topological invariants of knots and braid conjugacy classes, in the form of differential graded algebras, and present an explicit combinatorial formulation for these invariants. The algebras conjecturally give the relative contact homology of certain Legendrian tori in five-dimensional contact manifolds. We present several computations and derive a relation between the knot invariant and the Alexander polynomial.


Abstract: We provide a translation between Chekanov’s combinatorial theory for invariants of Legendrian knots in the standard contact $\mathbb{R}^3$ and a relative version of Eliashberg and Hofer’s Contact Homology. We use this translation to transport the idea of “coherent orientations” from the Contact Homology world to Chekanov’s combinatorial setting. As a result, we obtain a lifting of Chekanov’s differential graded algebra invariant to an algebra over $\mathbb{Z}[t, t^{-1}]$ with a full $\mathbb{Z}$ grading.

• Computable Legendrian invariants, *Topology* 42 (2003), no. 1, 55–82.

Abstract: We establish tools to facilitate the computation and application of the Chekanov-Eliashberg differential graded algebra (DGA), a Legendrian-isotopy invariant of Legendrian knots in standard contact three-space. More specifically, we reformulate the DGA in terms of front projection, and introduce the characteristic algebra, a new invariant derived from the DGA. We use our techniques to distinguish between several previously indistinguishable Legendrian knots and links.

  Abstract: We compute the maximal Thurston–Bennequin number for a Legendrian two-bridge knot or oriented two-bridge link in standard contact $\mathbb{R}^3$, by showing that the upper bound given by the Kauffman polynomial is sharp. As an application, we present a table of maximal Thurston–Bennequin numbers for prime knots with nine or fewer crossings.


  Summary: A connection discovered between the Heisenberg ferromagnet model and a problem in the theory of random walks allows us to verify the Bethe *ansatz* from physics in a special case, and to apply this case to solve the random walks problem.

• The rook on the half-chessboard, or how not to diagonalize a matrix (with K. Kedlaya), *Amer. Math. Monthly* 105 (1998), 819–824.


• $k$-ordered hamiltonian graphs (with M. Schultz), *J. Graph Theory* 24 (1997), 45–57.