THE LEGENDRIAN KNOT ATLAS

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This is the Legendrian knot atlas, available online at

`http://www.math.duke.edu/~ng/atlas/`

(permanent address: `http://alum.mit.edu/www/ng/atlas/`), and intended to accompany the paper "An atlas of Legendrian knots" by the authors [2]. This file was last changed on 14 August 2013.

The table on the following pages depicts conjectural classifications of Legendrian knots in all prime knot types of arc index up to 9. For each knot, we present a conjecturally complete list of non-destabilizable Legendrian representatives, modulo the symmetries of orientation reversal $L \mapsto -L$ and Legendrian mirroring $L \mapsto \mu(L)$. As usual, rotate 45° counterclockwise to translate from grid diagrams to fronts.

Each knot also comes with its conjectural Legendrian mountain range (extending infinitely downwards), comprised of black and red dots, plotted according to their Thurston–Bennequin number (vertical) and rotation number (horizontal). Arrows represent positive and negative Legendrian stabilization. The values of (tb, r) are not labeled but can be deduced from the values given for the non-destabilizable representatives. Boxes surround values of (tb, r) that have, or appear to have, more than one Legendrian representative, and mountain ranges without boxes represent knot types that are conjecturally Legendrian simple. The dots represent conjecturally distinct Legendrian isotopy classes; black dots are provably distinct classes, while red dots are conjecturally but not provably distinct from the black dots and each other. Thus the black dots represent a lower bound for the Legendrian mountain range, and the totality of dots represent our current best guess for the precise mountain range (which however could theoretically be larger or smaller than what is depicted).

Legendrian knots have been classified for several knot types, including torus knots and 4_1 [3] and twist knots [4]. These comprise the knots 3_1, 4_1, 5_1, 5_2, 6_1, 7_1, 7_2 in the table, along with their mirrors; for these knots, the mountain ranges depicted in the atlas agree with the classification results. In the table, we indicate torus knots by $T(p, q)$ and twist knots by K_n (for the knot with n half-twists, with the convention of [4]).

Using symmetries, we can produce from any Legendrian knot L up to four possibly distinct Legendrian knots: L, $-L$, $\mu(L)$, and $-\mu(L)$. The table depicts one representative from each of these orbits of up to four knots, along with information about which of the four knots in the orbit are isotopic, if any. For knots with nonzero rotation number, we choose a representative L

with positive rotation number, and L is trivially distinct from $-L$ and $\mu(L)$ (this fact depicted by hyphens in the table).

Grid diagrams labeled with matching letters (see e.g. 6_2) mark Legendrian knots that we believe but cannot yet prove to be distinct. For each depicted knot L, we also indicate which of L, $-L$, $\mu(L)$, and $-\mu(L)$ are Legendrian isotopic:

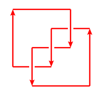
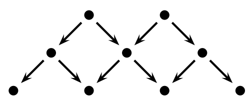
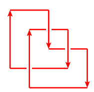
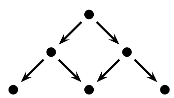
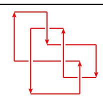
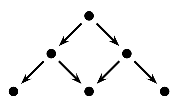
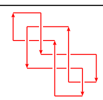
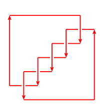
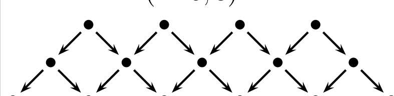
- ✓: the two knots have been verified by computer to be isotopic;
- ✗: the knots are provably distinct (see below);
- ✗?: we believe but cannot prove that the knots are distinct.

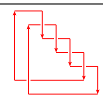
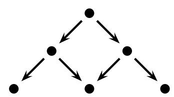
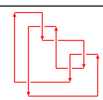
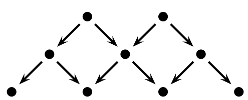
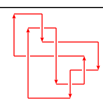
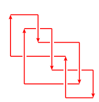
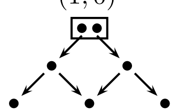
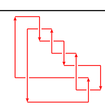
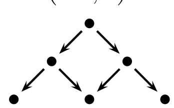
For instance, for $m(9_{42})$, let L denote the Legendrian knot depicted in the table. The computer program can show that $L = -\mu(L)$ but guesses that these are distinct from $-L = \mu(L)$. In the top row of the mountain range for $m(9_{42})$, the black dot represents one of these knots, and the red dot the other.

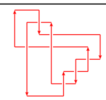
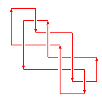
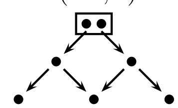
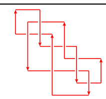
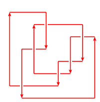
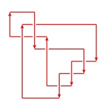
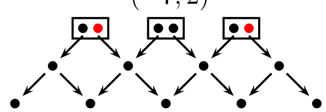
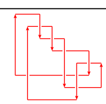
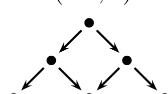
The techniques used to distinguish knots include two nonclassical Legendrian invariants, the graded ruling invariant [6] and the set of (Poincaré polynomials for) linearized contact homologies [1], which have been computed, where relevant, using the *Mathematica* notebook [5]. (Knots with no graded rulings/augmentations are denoted in these columns by a hyphen, for nonzero rotation number, or $\emptyset$, for zero rotation number.) For Legendrian knots that we have succeeded in distinguishing by means besides these invariants, please see [2, Section 3] for documentation.

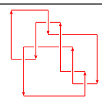
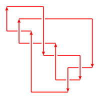
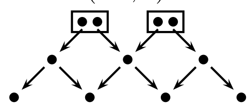
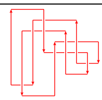
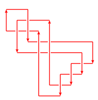
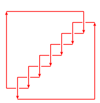
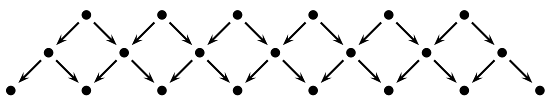
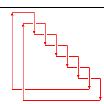
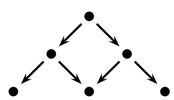
For some knots, the atlas omits a bit of information necessary to deduce a complete (conjectural) Legendrian classification, namely which Legendrian knots with the same (tb, r) stabilize to isotopic knots. This information is presented in Table 2, which follows the atlas. The knots given in Table 2 are those where there is some ambiguity about isotopy classes after stabilization; for all of those knots, the program guesses that the relevant Legendrian representatives either become isotopic after one (positive or negative) stabilization, or remain nonisotopic after arbitrarily many stabilizations.

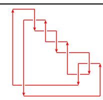
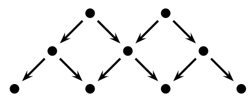
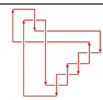
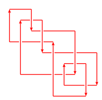
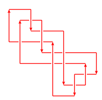
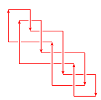
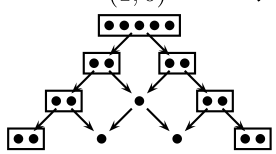
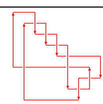
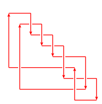
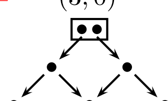
TABLE 1. Atlas of Legendrian Knots up to arc index 9

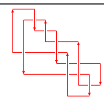
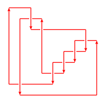
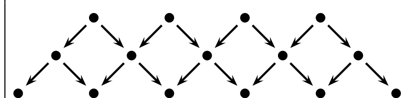
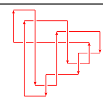
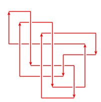
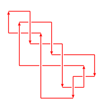
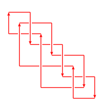
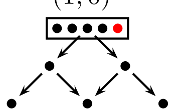
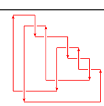
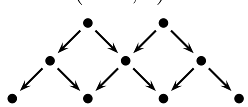
Knot Type	Grid Diagram	(tb, r)	$L = -L$?	$L = \mu(L)$?	$L = -\mu(L)$?	Ruling Invariant	Linearized Contact Homology	Note
3_1	 	$(-6, 1)$	-	-	✓	-	-	$T(2,-3)$, K_1
$m(3_1)$	 	$(1, 0)$	✓	✓	✓	$2 + z^2$	$2 + t$	$T(2,3)$, K_{-2}
$4_1 = m(4_1)$	 	$(-3, 0)$	✓	✓	✓	1	$t^{-1} + 2t$	$K_2 = K_{-3}$
5_1		$(-10, 1)$	-	-	✓	-	-	$T(2,-5)$
	 	$(-10, 3)$	-	-	✓	-	-	

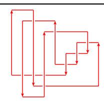
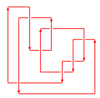
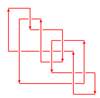
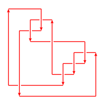
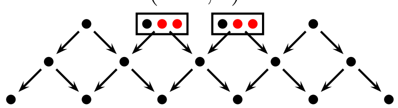
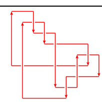
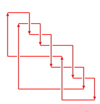
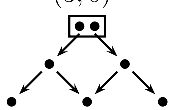
Knot Type	Grid Diagram	(tb, r)	$L = -L$?	$L = \mu(L)$?	$L = -\mu(L)$?	Ruling Invariant	Linearized Contact Homology	Note
$m(5_1)$		$(3, 0)$ 	✓	✓	✓	$3 + 4z^2 + z^4$	$4 + t$	$T(2, 5)$
5_2		$(-8, 1)$  †	-	-	✓	-	-	K_3
$m(5_2)$		$(1, 0)$	✓	✓	✓	1	$t^{-2} + t + t^2$	K_{-4}
		$(1, 0)$ 	✓	✓	✓	$1 + z^2$	$2 + t$	
6_1		$(-5, 0)$ 	✓	✓	✓	1	$2t^{-1} + 3t$	K_4

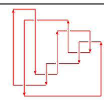
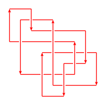
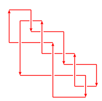
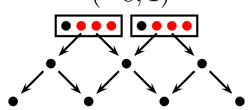
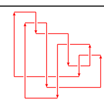
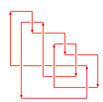
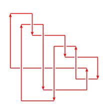
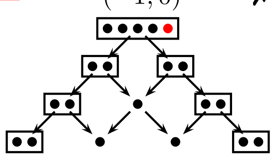
Knot Type	Grid Diagram	(tb, r)	$L = -L$?	$L = \mu(L)$?	$L = -\mu(L)$?	Ruling Invariant	Linearized Contact Homology	Note
$m(6_1)$		$(-3, 0)$	✓	✓	✓	1	$t^{-3} + t + t^3$	K_{-5}
		$(-3, 0)$	✓	✓	✓	1	$t^{-1} + 2t$	
								
6_2		$(-7, 0)$	✓	✗	✗	$\emptyset$	$\emptyset$	
	 a	$(-7, 2)$	-	-	✓	-	-	
	 a	$(-7, 2)$	-	-	✓	-	-	
								
$m(6_2)$		$(-1, 0)$	✓	✓	✓	$2 + z^2$	$t^{-1} + 2 + 2t$	
								

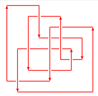
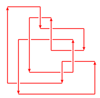
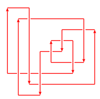
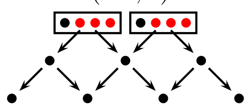
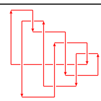
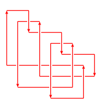
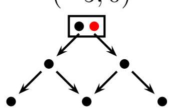
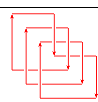
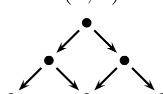
Knot Type	Grid Diagram	(tb, r)	$L = -L$?	$L = \mu(L)$?	$L = -\mu(L)$?	Ruling Invariant	Linearized Contact Homology	Note
$6_3 = m(6_3)$		$(-4, 1)$	-	-	✓	-	-	
		$(-4, 1)$	-	-	✓	-	-	
								
7_1		$(-14, 1)$	-	-	✓	-	-	$T(2, -7)$
		$(-14, 3)$	-	-	✓	-	-	
		$(-14, 5)$	-	-	✓	-	-	
								
$m(7_1)$		$(5, 0)$	✓	✓	✓	$4 + 10z^2 + 6z^4 + z^6$	$6 + t$	$T(2, 7)$
								

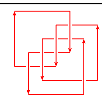
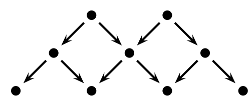
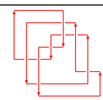
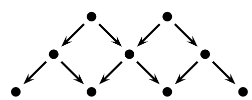
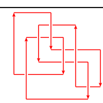
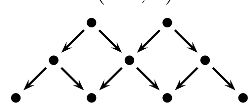
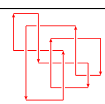
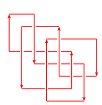
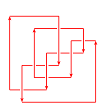
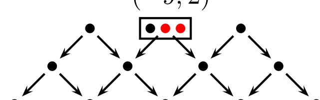
Knot Type	Grid Diagram	(tb, r)	$L = -L$?	$L = \mu(L)$?	$L = -\mu(L)$?	Ruling Invariant	Linearized Contact Homology	Note
7_2		$(-10, 1)$	-	-	✓	-	-	K_5
								
$m(7_2)$		$(1, 0)$	✓	✓	✓	1	$t^{-4} + t + t^4$	K_{-6}
		$(1, 0)$	✓	✓	✓	$1 + z^2$	$2 + t$	
		$(1, 0)$	✗	✗	✓	1	$t^{-2} + t + t^2$	
		$(1, 0)$	✓	✓	✓	$1 + z^2$	$2 + t$	
								
7_3		$(3, 0)$	✓	✓	✓	1	$2t^{-2} + t + 2t^2$	
		$(3, 0)$	✓	✓	✓	$1 + 3z^2 + z^4$	$4 + t$	
								

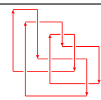
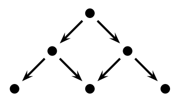
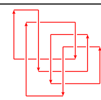
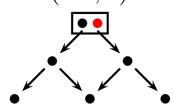
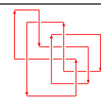
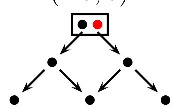
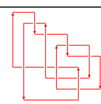
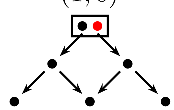
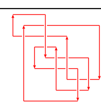
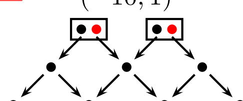
Knot Type	Grid Diagram	(tb, r)	$L = -L$?	$L = \mu(L)$?	$L = -\mu(L)$?	Ruling Invariant	Linearized Contact Homology	Note
$m(7_3)$		$(-12, 1)$	-	-	✓	-	-	
		$(-12, 3)$	-	-	✓	-	-	
								
7_4	 b	$(1, 0)$	✓	✓	✓	$\emptyset$	$\emptyset$	
	 b	$(1, 0)$	✓	✓	✓	$\emptyset$	$\emptyset$	
		$(1, 0)$	✓	✗	✗	$\emptyset$	$\emptyset$	
		$(1, 0)$	✓	✓	✓	z^2	$2 + t$	
								
$m(7_4)$		$(-10, 1)$	-	-	✓	-	-	
								

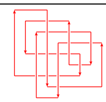
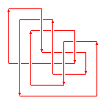
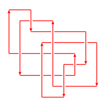
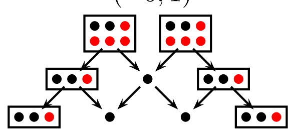
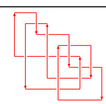
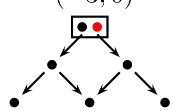
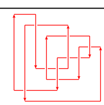
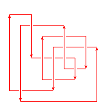
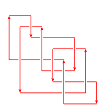
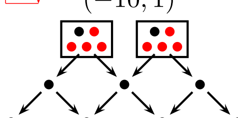
Knot Type	Grid Diagram	(tb, r)	$L = -L$?	$L = \mu(L)$?	$L = -\mu(L)$?	Ruling Invariant	Linearized Contact Homology	Note
7_5	 c	$(-12, 1)$	-	-	✓	-	-	
	 c	$(-12, 1)$	-	-	✓	-	-	
	 c	$(-12, 1)$	-	-	✓	-	-	
		$(-12, 3)$	-	-	✓	-	-	
								
$m(7_5)$		$(3, 0)$	✓	✓	✓	$2 + z^2$	$t^{-2} + 2 + t + t^2$	
		$(3, 0)$	✓	✓	✓	$2 + 3z^2 + z^4$	$4 + t$	
								

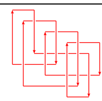
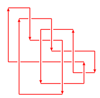
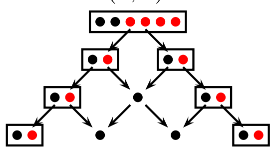
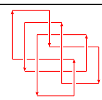
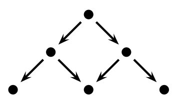
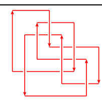
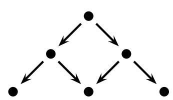
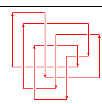
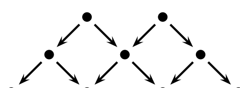
Knot Type	Grid Diagram	(tb, r)	$L = -L$?	$L = \mu(L)$?	$L = -\mu(L)$?	Ruling Invariant	Linearized Contact Homology	Note
7_6	 d	$(-8, 1)$	-	-	✓	-	-	
	 d	$(-8, 1)$	-	-	✓	-	-	
	 d	$(-8, 1)$	-	-	✗?	-	-	
								
$m(7_6)$		$(-1, 0)$	✓	✓	✓	$1 + z^2$	$t^{-1} + 2 + 2t$	
		$(-1, 0)$	✗?	✗?	✓	$1 + z^2$	$t^{-1} + 2 + 2t$	
		$(-1, 0)$	✗	✗	✓	1	$t^{-2} + t^{-1} + 2t + t^2$	
								

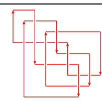
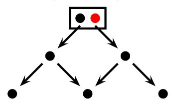
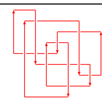
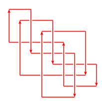
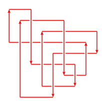
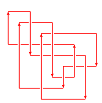
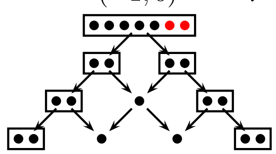
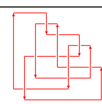
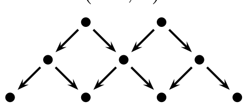
Knot Type	Grid Diagram	(tb, r)	$L = -L$?	$L = \mu(L)$?	$L = -\mu(L)$?	Ruling Invariant	Linearized Contact Homology	Note
7_7	 e	$(-4, 1)$	-	-	✗?	-	-	
	 e	$(-4, 1)$	-	-	✓	-	-	
	 e	$(-4, 1)$	-	-	✓	-	-	
								
$m(7_7)$	 f	$(-5, 0)$	✓	✓	✓	1	$2t^{-1} + 3t$	
	 f	$(-5, 0)$	✓	✓	✓	1	$2t^{-1} + 3t$	
								
8_{19}		$(5, 0)$	✓	✓	✓	$5 + 10z^2 + 6z^4 + z^6$	$6 + t$	$T(3, 4)$
								

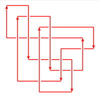
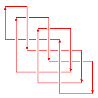
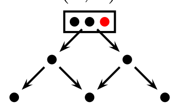
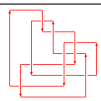
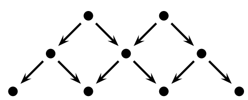
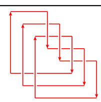
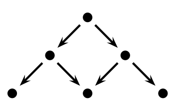
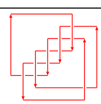
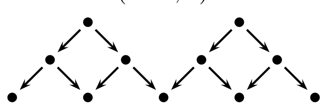
Knot Type	Grid Diagram	(tb, r)	$L = -L$?	$L = \mu(L)$?	$L = -\mu(L)$?	Ruling Invariant	Linearized Contact Homology	Note
$m(8_{19})$		$(-12, 1)$ 	-	-	✓	-	-	$T(3,-4)$
8_{20}		$(-6, 1)$ 	-	-	✓	-	-	
$m(8_{20})$		$(-2, 1)$ 	-	-	✓	-	-	
8_{21}	 g	$(-9, 0)$	✓	✓	✓	$\emptyset$	$\emptyset$	
	 g	$(-9, 0)$	✓	✗?	✗?	$\emptyset$	$\emptyset$	
		$(-9, 2)$ 	-	-	✓	-	-	

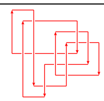
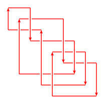
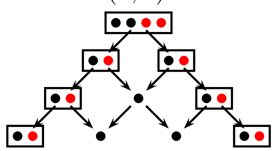
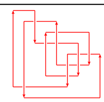
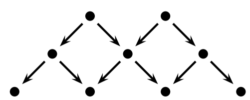
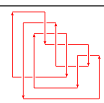
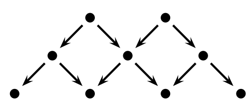
Knot Type	Grid Diagram	(tb, r)	$L = -L$?	$L = \mu(L)$?	$L = -\mu(L)$?	Ruling Invariant	Linearized Contact Homology	Note
$m(8_{21})$		$(1, 0)$ 	✓	✓	✓	$3 + 2z^2$	$2 + t,\ t^{-1} + 4 + 2t$	
9_{42}		$(-3, 0)$ 	✓	✗?	✗?	$2 + z^2$	$2t^{-1} + 2 + 3t$	
$m(9_{42})$		$(-5, 0)$ 	✗?	✗?	✓	$\emptyset$	$\emptyset$	
9_{43}		$(1, 0)$ 	✗?	✗?	✓	$3 + 4z^2 + z^4$	$t^{-1} + 4 + 2t$	
$m(9_{43})$		$(-10, 1)$ 	-	-	✗?	-	-	

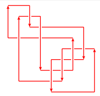
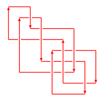
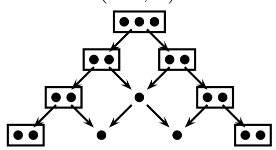
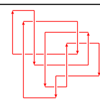
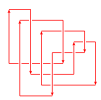
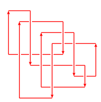
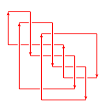
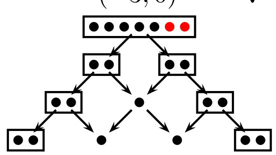
Knot Type	Grid Diagram	(tb, r)	$L = -L$?	$L = \mu(L)$?	$L = -\mu(L)$?	Ruling Invariant	Linearized Contact Homology	Note
9_{44}		$(-6, 1)$	-	-	✗?	-	-	
	 h	$(-6, 1)$	-	-	✗?	-	-	
	 h	$(-6, 1)$	-	-	✗?	-	-	
								
$m(9_{44})$		$(-3, 0)$	✗?	✓	✗?	1	$t^{-1} + 2t$	
								
9_{45}	 i	$(-10, 1)$	-	-	✗?	-	-	
	 i	$(-10, 1)$	-	-	✓	-	-	
	 i	$(-10, 1)$	-	-	✗?	-	-	
								

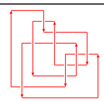
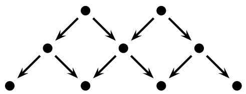
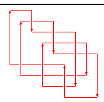
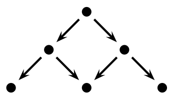
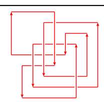
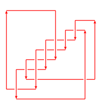
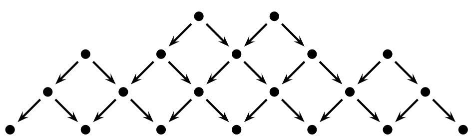
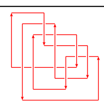
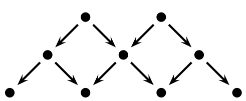
Knot Type	Grid Diagram	(tb, r)	$L = -L$?	$L = \mu(L)$?	$L = -\mu(L)$?	Ruling Invariant	Linearized Contact Homology	Note
$m(9_{45})$		$(1, 0)$	✗?	✗?	✗?	$2 + 2z^2$	$2 + t,\ t^{-1} + 4 + 2t$	
		$(1, 0)$	✓	✗?	✗?	$2 + z^2$	$2 + t,\ t^{-2} + t^{-1} + 2 + 2t + t^2$	
								
9_{46}		$(-7, 0)$	✓	✓	✓	1	$3t^{-1} + 4t$	
								
$m(9_{46})$		$(-1, 0)$	✓	✓	✓	2	t	
								
9_{47}		$(-2, 1)$	-	-	✓	-	-	
								

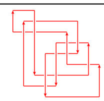
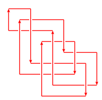
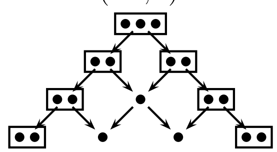
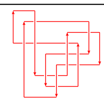
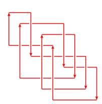
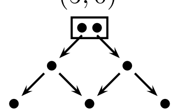
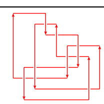
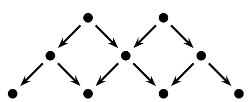
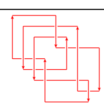
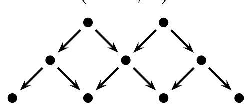
Knot Type	Grid Diagram	(tb, r)	$L = -L$?	$L = \mu(L)$?	$L = -\mu(L)$?	Ruling Invariant	Linearized Contact Homology	Note
$m(9_{47})$	 	$(-7, 0)$	✗?	✗?	✓	1	$3t^{-1} + 4t$	
9_{48}		$(-1, 0)$	✓	✓	✓	z^2	$t^{-1} + 2 + 2t$	
		$(-1, 0)$	✗?	✗?	✓	z^2	$t^{-1} + 2 + 2t$	
		$(-1, 0)$	✗	✗	✓	$\emptyset$	$\emptyset$	
	 	$(-1, 0)$	✓	✗?	✗?	$\emptyset$	$\emptyset$	
$m(9_{48})$	 	$(-8, 1)$	-	-	✓	-	-	

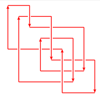
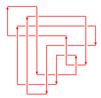
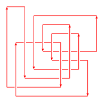
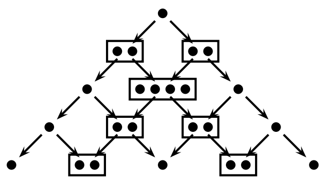
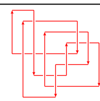
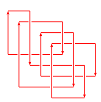
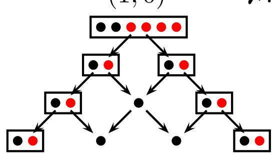
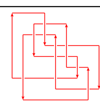
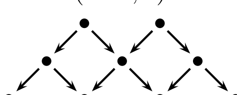
Knot Type	Grid Diagram	(tb, r)	$L = -L$?	$L = \mu(L)$?	$L = -\mu(L)$?	Ruling Invariant	Linearized Contact Homology	Note
9_{49}		$(3, 0)$	✓	✗?	✗?	$\emptyset$	$\emptyset$	
		$(3, 0)$ 	✓	✓	✓	$2z^2 + z^4$	$4 + t$	
$m(9_{49})$		$(-12, 1)$ 	-	-	✓	-	-	
10_{124}		$(7, 0)$ 	✓	✓	✓	$7 + 21z^2 + 21z^4 + 8z^6 + z^8$	$8 + t$	$T(3, 5)$
$m(10_{124})$		$(-15, 2)$ 	-	-	✓	-	-	$T(3, -5)$

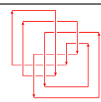
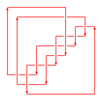
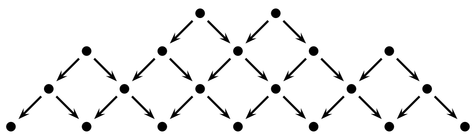
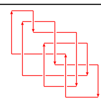
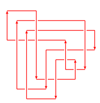
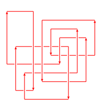
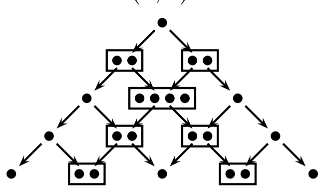
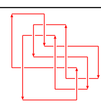
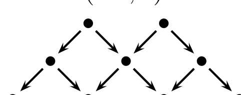
Knot Type	Grid Diagram	(tb, r)	$L = -L$?	$L = \mu(L)$?	$L = -\mu(L)$?	Ruling Invariant	Linearized Contact Homology	Note
10_{128}		$(5, 0)$	✓	✗?	✗?	$2 + z^2$	$2t^{-2} + 2 + t + 2t^2$	
		$(5, 0)$	✗?	✓	✗?	$2 + 6z^2 + 5z^4 + z^6$	$6 + t$	
								
$m(10_{128})$		$(-14, 1)$	-	-	✓	-	-	
								
10_{132}		$(-8, 1)$	-	-	✓	-	-	
								

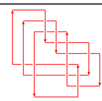
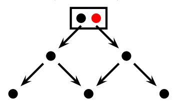
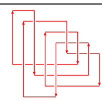
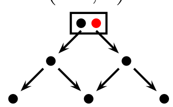
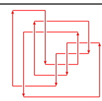
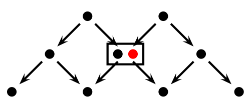
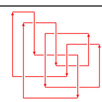
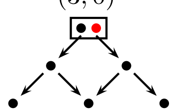
Knot Type	Grid Diagram	(tb, r)	$L = -L$?	$L = \mu(L)$?	$L = -\mu(L)$?	Ruling Invariant	Linearized Contact Homology	Note
$m(10_{132})$		$(-1, 0)$	✗	✗	✓	$\emptyset$	$\emptyset$	
		$(-1, 0)$	✓	✓	✓	$\emptyset$	$\emptyset$	
								
10_{136}		$(-3, 0)$	✗	✗	✓	1	$t^{-2} + 2t^{-1} + 3t + t^2$	
		$(-3, 0)$	✓	✓	✓	1	$t^{-2} + 2t^{-1} + 3t + t^2$	
		$(-3, 0)$	✓	✗?	✗?	$1 + z^2$	$2t^{-1} + 2 + 3t$	
		$(-3, 0)$	✓	✗?	✗?	$1 + z^2$	$2t^{-1} + 2 + 3t$	
								

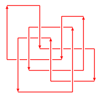
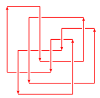
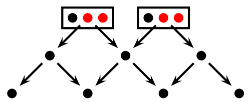
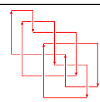
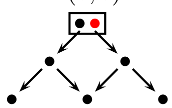
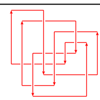
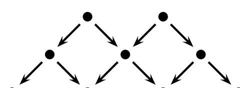
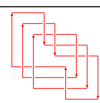
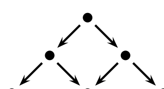
Knot Type	Grid Diagram	(tb, r)	$L = -L$?	$L = \mu(L)$?	$L = -\mu(L)$?	Ruling Invariant	Linearized Contact Homology	Note
$m(10_{136})$	 	$(-6, 1)$	-	-	✓	-	-	
10_{139}	 	$(7, 0)$	✓	✓	✓	$6 + 21z^2 + 21z^4 + 8z^6 + z^8$	$8 + t$	
$m(10_{139})$		$(-16, 1)$	-	-	✓	-	-	
	 	$(-17, 4)$	-	-	✓	-	-	
10_{140}	 	$(-8, 1)$	-	-	✓	-	-	

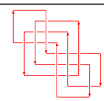
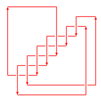
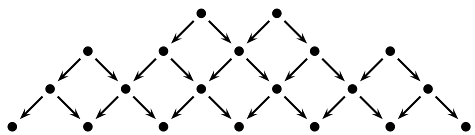
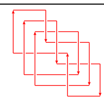
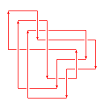
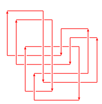
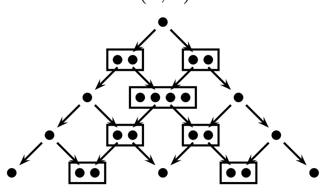
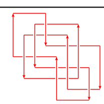
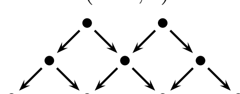
Knot Type	Grid Diagram	(tb, r)	$L = -L$?	$L = \mu(L)$?	$L = -\mu(L)$?	Ruling Invariant	Linearized Contact Homology	Note
$m(10_{140})$		$(-1, 0)$	✗	✗	✓	1	t	
	 	$(-1, 0)$	✓	✓	✓	1	t	
10_{142}		$(5, 0)$	✓	✓	✓	1	$3t^{-2} + t + 3t^2$	
	 	$(5, 0)$	✓	✓	✓	$1 + 6z^2 + 5z^4 + z^6$	$6 + t$	
$m(10_{142})$	 	$(-14, 1)$	-	-	✓	-	-	
10_{145}	 	$(-12, 1)$	-	-	✓	-	-	

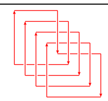
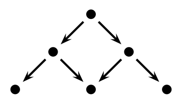
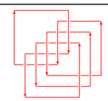
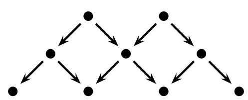
Knot Type	Grid Diagram	(tb, r)	$L = -L$?	$L = \mu(L)$?	$L = -\mu(L)$?	Ruling Invariant	Linearized Contact Homology	Note
$m(10_{145})$		$(3, 0)$	✓	✓	✓	$2 + 4z^2 + z^4$	$4 + t$	
		$(2, 1)$	-	-	✓	-	-	
		$(1, 0)$	✓	✓	✓	$\emptyset$	$\emptyset$	
								
10_{160}		$(1, 0)$	✓	✗?	✗?	1	$2t^{-2} + t^{-1} + 2t + 2t^2$	
		$(1, 0)$	✗?	✗?	✗?	$1 + 3z^2 + z^4$	$t^{-1} + 4 + 2t$	
								
$m(10_{160})$		$(-10, 1)$	-	-	✓	-	-	
								

Knot Type	Grid Diagram	(tb, r)	$L = -L$?	$L = \mu(L)$?	$L = -\mu(L)$?	Ruling Invariant	Linearized Contact Homology	Note
10_{161}		$(-14, 1)$	-	-	✓	-	-	
		$(-15, 4)$	-	-	✓	-	-	
								
$m(10_{161})$		$(5, 0)$	✓	✓	✓	$3 + 9z^2 + 6z^4 + z^6$	$6 + t$	
		$(4, 1)$	-	-	✓	-	-	
		$(3, 0)$	✓	✓	✓	$\emptyset$	$\emptyset$	
								
$11n_{19}$		$(-8, 1)$	-	-	✓	-	-	
								

Knot Type	Grid Diagram	(tb, r)	$L = -L$?	$L = \mu(L)$?	$L = -\mu(L)$?	Ruling Invariant	Linearized Contact Homology	Note
$m(11n_{19})$		$(-1, 0)$ 	✓	✗?	✗?	$3 + 4z^2 + z^4$	$2t^{-1} + 4 + 3t$	
$11n_{38}$		$(-5, 0)$ 	✓	✗?	✗?	$2 + z^2$	$3t^{-1} + 2 + 4t$	
$m(11n_{38})$		$(-4, 1)$ 	-	-	✓	-	-	
$11n_{95}$		$(3, 0)$ 	✓	✗?	✗?	$3 + 6z^2 + 2z^4$	$4 + t,\ t^{-1} + 6 + 2t$	

Knot Type	Grid Diagram	(tb, r)	$L = -L$?	$L = \mu(L)$?	$L = -\mu(L)$?	Ruling Invariant	Linearized Contact Homology	Note
$m(11n_{95})$	 j	$(-12, 1)$	-	-	✗?	-	-	
	 j 	$(-12, 1)$	-	-	✓	-	-	
$11n_{118}$	 	$(3, 0)$	✗?	✗?	✓	$4 + 7z^2 + 2z^4$	$4 + t$, $t^{-1} + 6 + 2t$	
$m(11n_{118})$	 	$(-12, 1)$	-	-	✓	-	-	
$12n_{242}$	 	$(9, 0)$	✓	✓	✓	$9 + 39z^2 + 57z^4 + 36z^6 + 10z^8 + z^{10}$	$10 + t$	

Knot Type	Grid Diagram	(tb, r)	$L = -L$?	$L = \mu(L)$?	$L = -\mu(L)$?	Ruling Invariant	Linearized Contact Homology	Note
$m(12n_{242})$		$(-18, 1)$	-	-	✓	-	-	
		$(-19, 4)$	-	-	✓	-	-	
								
$12n_{591}$		$(7, 0)$	✓	✓	✓	$4 + 17z^2 + 20z^4 + 8z^6 + z^8$	$8 + t$	
		$(6, 1)$			✓			
		$(5, 0)$	✓	✓	✓	$\emptyset$	$\emptyset$	
								
$m(12n_{591})$		$(-16, 1)$	-	-	✓	-	-	
								

Knot Type	Grid Diagram	(tb, r)	$L = -L$?	$L = \mu(L)$?	$L = -\mu(L)$?	Ruling Invariant	Linearized Contact Homology	Note
$15n_{41185}$	 	$(11, 0)$	✓	✓	✓	$14 + 70z^2 + 133z^4 + 121z^6$ $+55z^8 + 12z^{10} + z^{12}$	$12 + t$	$T(4, 5)$
$m(15n_{41185})$	 	$(-20, 1)$	-	-	✓	-	-	$T(4, -5)$

Knot	Isotopy classes after S_+	Isotopy classes after S_-
$m(7_2)$	$L_1, L_2, -L_3 \mid L_3, L_4$	$L_1, L_2, L_3 \mid -L_3, L_4$
$m(7_6)$	$L_1, L_3 \mid L_2, -L_2, -L_3$	$L_1, -L_3 \mid L_2, -L_2, L_3$
9_{44}	$L_1, -\mu(L_1) \mid L_2, -\mu(L_3) \ : \ -\mu(L_2), L_3$	$-L_1, \mu(L_1) \mid -L_2, \mu(L_3) \ : \ \mu(L_2), -L_3$
$m(9_{45})$	$L_1, \mu(L_1), \mu(L_2) \ : \ -L_1, -\mu(L_1), L_2$	$L_1, \mu(L_1), L_2 \ : \ -L_1, -\mu(L_1), \mu(L_2)$
9_{48}	$L_1, L_3 \mid L_2, -L_2, -L_3, L_4, \mu(L_4)$	$L_1, -L_3 \mid L_2, -L_2, L_3, L_4, \mu(L_4)$
10_{128}	$L_1, -L_2 \ : \ \mu(L_1), L_2$	$L_1, L_2 \ : \ \mu(L_1), -L_2$
$m(10_{132})$	$L_1 \mid -L_1, L_2$	$L_1, L_2 \mid -L_1$
10_{136}	$L_1, L_4, \mu(L_4) \mid -L_1, L_2, L_3, \mu(L_3)$	$L_1, L_2, L_3, \mu(L_3) \mid -L_1, L_4, \mu(L_4)$
$m(10_{140})$	$L_1 \mid -L_1, L_2$	$L_1, L_2 \mid -L_1$
$m(10_{145})$	$S_-(L_1) \mid -L_2, L_3$	$S_+(L_1) \mid L_2, L_3$
10_{160}	$L_1, L_2, \mu(L_2) \ : \ \mu(L_1), -L_2, -\mu(L_2)$	$L_1, -L_2, -\mu(L_2) \ : \ \mu(L_1), L_2, \mu(L_2)$
$m(10_{161})$	$S_-(L_1) \mid -L_2, L_3$	$S_+(L_1) \mid L_2, L_3$
$12n_{591}$	$S_-(L_1) \mid -L_2, L_3$	$S_+(L_1) \mid L_2, L_3$

TABLE 2. Information about isotopy classes of Legendrian knots after stabilization. For each knot type, $L_1, L_2, \ldots$ denote the Legendrian knots depicted in the atlas, ordered from top to bottom. In this table, knots separated by commas can be shown to be Legendrian isotopic after one application of the appropriate stabilization. Vertical bars separate knots that are provably distinct after any number of the appropriate stabilizations; colons separate knots that the program conjectures are distinct after any number of stabilizations.

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