Part of Hints for Hw 9

Math 321

By Lei Li

Some easy problems

1. a). $(0 \le \rho \le 2, 0 \le \theta \le 2\pi, 0 \le z \le 4)$
b). $dV = \rho d\rho d\theta dz$
c). $\frac{256\pi}{3}$

2. a). $\frac{25}{3}$
b). 3

1.5

#2. For area, use dS_r $Area = \int_S R^2 \sin\theta d\theta d\varphi = \int_0^{2\pi} d\varphi \int_0^{\pi} R^2 \sin\theta d\theta = 4\pi R^2$
Volume: $V = \int_{ball} r^2 \sin\theta dr d\theta d\varphi = \int_0^R dr \int_0^{2\pi} d\varphi \int_0^{\pi} r^2 \sin\theta d\theta = \frac{4}{3}\pi R^3$
#4. $\vec{r}(r,\theta,\varphi) = (R + r\cos\theta)\cos\varphi\vec{e}_x + (R + r\cos\theta)\sin\varphi\vec{e}_y + r\sin\theta\vec{e}_z$. where $0 \le r \le a$ and $0 \le \theta, \varphi \le 2\pi$
$dV = r(R + r\cos\theta)dr d\theta d\varphi$. The volume is $2\pi^2 Ra^2$

1.6

#1. The answers would be the same as in 1.5. However, the methods are a little different. You should calculate h_i first. However, $d\vec{S} = \vec{N}dq_i dq_j$ still. The only difference is how to calculate $\vec{N}$ etc. I don't want to list the detail.
#2. $\vec{r} = r\hat{r}$. For the curve, $\vec{r} = \rho(\theta)\hat{r}$. $d\vec{r} = \rho'(\theta)d\theta\hat{r} + \rho(\theta)d\theta\hat{\theta}$. $L = \int_\alpha^\beta \sqrt{\rho'(\theta)^2 + \rho^2}d\theta$.
For the area, $0 \le r \le \rho(\theta)$. We can get $\frac{\partial\vec{r}}{\partial r} = 1\hat{r}$ and $\frac{\partial\vec{r}}{\partial\theta} = r\hat{\theta}$. $dS = rdrd\theta$. $A = \int_\alpha^\beta \frac{1}{2}\rho^2 d\theta$.
#3. Omitted. You can get the answers by yourself.